Name:

MATH 332-001, APRIL, 2010,	MIDTERM

Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Closed book, open mind, one page front and back of notes allowed. Any calculator also allowed, no cell phones or laptops. Thanks and enjoy. You have 2 hours to attempt this exam. There are 3000pts required points to earn on this exam.

Problem 1 [300pts] Calculate or answer question (may be multiple answers, you just give a correct one)

(a.) Find f' if $f(x, y) = xy$

(b.) Find g' if $g(t) = (t, t^2)$

(c.) Is $f(x, y, z) = (x + 2y, y + z, 3x + z)$ a linear mapping?

(d.) How is the angle between two vectors in $\mathbb{R}^n$ defined?

(e.) Show $f(x, y) = (2y + 1, 3x - 2)$ is onto $\mathbb{R}^2$

Problem 2 [300pts] Use the implicit function (or mapping) or inverse function (or mapping) theorem to answer the questions below. Make sure to verify the interesting hypothesis for the theorem that you use, I will provide grace for the fine print,

(a.) Can $x^2 + y^2 + z^2 = 6$ be solved for z as a function of (x, y) in an open ball centered at $(1, 1, 2)$?

(b.) Can $x^2 + y^2 + z^2 = 6$ be solved for z as a function of (x, y) in an open ball centered at $(1, 1, 1)$?

(c.) Does the implicit function theorem yield that $x^2 + y^2 = 4$ be solved for y as a function of x in an open disk centered at $(2, 0)$?

(d.) Where is the mapping $F(x, y) = (e^{(x-1)^2}, x + e^{(y-1)^2})$ locally invertible?

Problem 3 [300pts] Suppose $\gamma(t) = (\cos(t), \sin(t), t)$ for $t \geq 0$. Calculate T, N and B for the given curve. Also, find the arclength as a function of t.

Problem 4 [300pts] Given that $xyz + \cos(x^2 + z^2) = w$ calculate

$$\left(\frac{\partial w}{\partial y}\right)_{x,z} \qquad \text{and} \qquad \left(\frac{\partial y}{\partial w}\right)_{x,z}.$$

Problem 5 [300pts] Suppose that

$$x^2 + y + z + e^w = 2 \qquad \text{and} \qquad x + y^2 + z + w^3 = 0.$$

Calculate $\left(\frac{\partial w}{\partial x}\right)_z$. State which variables are dependent and which are independent at the start of your calculation.

Problem 6 [300pts] Find point(s) on the circle $(x-1)^2 + (y-1)^2 = 1$ which are nearest the line $x + y = 8$. Use the method of Lagrange multipliers..

Problem 7 [300pts] Find the dimensions of the largest box which can be inscribed in the ellipsoid $x^2 + y^2 + \frac{z^2}{4} = 1$.

Problem 8 [300pts] Suppose $f(x, y) = xy$ find the multivariate Taylor series for f centered at $(2, 3)$.

Problem 9 [300pts] Suppose $g(t) = (t^2, \cos(t))$ and $F(x, y) = (xy, x^2, y^2)$. Calculate $(F \circ g)'(t)$.

Problem 10 [300pts] **Prove** that if $F_1, F_2 : \mathbb{R}^n \to \mathbb{R}^m$ are differentiable at $b \in \mathbb{R}^n$ then $F_1 + F_2$ is differentiable at $b \in \mathbb{R}^n$ and $(F_1 + F_2)'(b) = F_1'(b) + F_2'(b)$. You may assume the theorems concerning limits of mappings.

Bonus [150pts] Compare and contrast the directional derivative and the differential. Give an example of a mapping whose directional derivatives exist yet the differential does not exist. What important property is your example necessarily missing?