

(This is the coversheet for the homework. The actual solutions should be worked out neatly on single-sided with each problem labeled (for example: Problem 1, 1.1 #14 ) and the answer boxed. Those additional sheets are then to be stapled with a metal staple in the upper left corner in such a way that no work is obscured. Failure to follow the style outlined here will result in a loss of credit. The problems refer to Anton and Rorres 10th ed. of *Elementary Linear Algebra: applications version.* )

**Problem 1** § 1.1 # 14 (augmented matrix for system linear eqns.)

**Problem 2** § 1.2 # 6 (solve via Gauss-Jordan elimination)

**Problem 3** § 1.2 # 8 (solve via Gauss-Jordan elimination)

**Problem 4** § 1.2 # 30 (solve system in terms of unknown constants  $a, b, c$ )

**Problem 5** § 1.3 # 6 (computation of transpose, sum, difference and products and trace)

**Problem 6** § 1.3 # 10a (matrix-vector product as linear combination)

**Problem 7** § 1.3 # 12 (matrix notation for system of linear eqns.)

**Problem 8** Let  $A, B \in \mathbb{R}^{n \times n}$ . Let  $\text{trace}(A) = \sum_{i=1}^n A_{ii}$ .

- a. If  $c \in \mathbb{R}$ , show  $\text{tr}(A + cB) = \text{tr}(A) + c\text{tr}(B)$ .
- b. show that  $\text{trace}(AB) = \text{trace}(BA)$ .
- c. let  $[A, B] = AB - BA$ , show  $\text{trace}([A, B]) = 0$ .
- d. can  $[A, B] = I$  ?

**\*\*please use technology to compute matrix operations in the problems that follow.\*\***

**Problem 9** § 1.2 # 34 (substitution to make linear)

**Problem 10** Find a cubic polynomial whose graph contains the points  $(1, 2)$ ,  $(2, 2)$ ,  $(3, 2)$  and  $(4, 2)$ .

**Problem 11** § 1.2 # 38 (circles that fit the triple of points, find all solutions, answer has parameter)

**Problem 12** A **probability vector** has entries whose sum is one. A **stochastic matrix** is a square matrix whose columns are probability vectors. Let  $A = \begin{bmatrix} .85 & .03 \\ .15 & .97 \end{bmatrix}$  be a stochastic matrix which models the migration of people to and from the city to the suburbs. In particular, if  $c_k$  denotes the number of people (in thousands) in the city in year  $k$  and  $s_k$  denotes the number of people (in thousands) living in the suburbs in year  $k$  then letting  $X_k = [c_k, s_k]^T$  we can model the migration of people by the following matrix product:

$$X_{k+1} = AX_k.$$

This model assumes the population stays constant and that there are only two places to live, the city or the suburbs. In addition, the model says that 15% of city people move to suburbs while only 3% of the suburb people move to the city. Suppose that  $X_0 = [400, 300]^T$  represents the population in 2000. **Calculate the number of people living in the city and the number of people in suburbs in the years 2001, 2002, ... 2012. Give your answer in a table.**