(This is the coversheet for the homework. The actual solutions should be worked out neatly on single-sided with each problem labeled (for example: Problem 1, 1.1 #14) and the answer boxed. Those additional sheets are then to be stapled with a metal staple in the upper left corner in such a way that no work is obscured. Failure to follow the style outlined here will result in a loss of credit. The problems refer to Anton and Rorres 10th ed. of *Elementary Linear Algebra: applications version.*)

Problem 28 Suppose $M \in \mathbb{R}^{m\times m}$ and $N \in \mathbb{R}^{n\times n}$ such that x_1, x_2 are solutions $Mx_1 = y_1$ and $Nx_2 = y_2$. Let A be a block-diagonal matrix of the form:

$$A = \left[\begin{array}{c|c} M & 0 \\ \hline 0 & N \end{array}\right]$$

If $det(M), det(N) \neq 0$ then solve $Az = w$ where $w = [2y_1, 3y_2]^T$.

Problem 29 § 2.1 # 10 (arrow-technique to calculate 3×3 determinant)

Problem 30 § 2.1 # 16 (equation given by determinant, solve it)

Problem 31 § 2.1 # 32 (determinant by inspection)

Problem 32 § 2.2 # 12 (row-reduction to calculate det)

Problem 33 § 2.2 # 20 (properties of det calculation)

Problem 34 The matrix $A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ is an example of a **Vandermonde matrix**. Use properties about row and column operations to show that $det(A) = (b-a)(c-a)(c-b)$.

Problem 35 Define a **Vandermonde matrix** $V(t)$ as follows:

$$V(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \end{bmatrix}$$

This matrix provides us a convenient way of creating a cubic polynomial that passes through distinct zeros $(x_1, 0)$, $(x_2, 0)$, $(x_3, 0)$. Define $f(x) = det(V(x))$ and explain why $f(x_1) = f(x_2) = f(x_3) = 0$. Based on an analogy to the preceding problem state an explicit formula for $f(t)$.

Problem 36 § 2.3 # 8 (invertible? use det to decide)

Problem 37 § 2.3 # 18 (choose k to make matrix invertible)

Problem 38 Supplementary exercises, page 117 #26 (Cramer's rule for symbolic problem)

Problem 39 Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.

(a.) let $p(t) = det(A - tI)$. Calculate the formula for $p(t)$

(b.) show that $p(A) = 0$

Problem 40 Let $\vec{v}_1 = \langle 1,2,3\rangle$, $\vec{v}_2 = \langle 1,1,0\rangle$ and $\vec{v}_3 = \langle 2,1,3\rangle$. Find the volume of a parallel-piped with edges $\vec{v}_1, \vec{v}_3, \vec{v}_3$. Is $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ a right-handed triple?