(This is the coversheet for the homework. The problems refer to Anton and Rorres 10th ed. of *Elementary Linear Algebra: applications version.* See Problem Sets 1, 2 or 3 for further formatting.)

Problem 72 § 4.10 #4 (standard matrix, relation of composition and multiplication)

Problem 73 § 4.10 #6 (rotations, reflections, compositions thereof in $\mathbb{R}^2$)

Problem 74 § 4.10 #10 (rotations, reflections, compositions thereof in $\mathbb{R}^3$)

Problem 75 § 4.10 #14 (injective? If so, find inverse)

Problem 76 § 4.11 #4 (please sketch the transformed unit-square for each, let this suffice for words)

Problem 77 § 4.6 #2 (coordinate vectors relative to nonstandard basis of $\mathbb{R}^3$)

Problem 78 § 4.6 #4 (coordinate vectors for polynomials)

Problem 79 § 4.6 #6 (transition matrix for bases of $\mathbb{R}^2$)

Problem 80 Let $T(f(x)) = x^2 f''(x)$ define a linear transformation on P_2. Find the matrix of T with respect to the usual basis $\{1, x, x^2\}$ for P_2.

Problem 81 Let $V = span\{\cos(x), \sin(x), e^x\}$ and suppose $S, T : V \to V$ are defined by $T(f) = df/dx$ and $S(f) = \int_0^x f(t)\, dt$ for all $f \in V$. Find the matrices for S and T relative the natural basis $\beta = \{\cos(x), \sin(x), e^x\}$. Compute the product of the matrices $[S]_{\beta,\beta}$ and $[T]_{\beta,\beta}$ and comment on the significance of the result. *Hint: if you've taken calculus and you don't know this, I kill you.*

Problem 82 Suppose V denotes three-dimensional physical space. Furthermore, suppose $T : V \to V$ is a rotation by angle θ. Consider this, we can **choose** coordinates on V which make the axis of rotation the z-axis. It then follows that the standard-matrix of T has the form

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Suppose we choose any other basis for V, say β, and let $[T]_{\beta,\beta} = R$. Calculate $trace(R)$.

Problem 83 Suppose $R = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is a rotation matrix on $\mathbb{R}^3$. By what angle does R rotate? What is the axis of rotation? Find a vector in the plane which takes the axis as its normal and show that the vector rotates as claimed.

Note: the axis is not hard to find here if you think about how a rotation acts on the axis. There is a formula to find the axis in general for an arbitrary rotation matrix R. See #28 of § 4.9