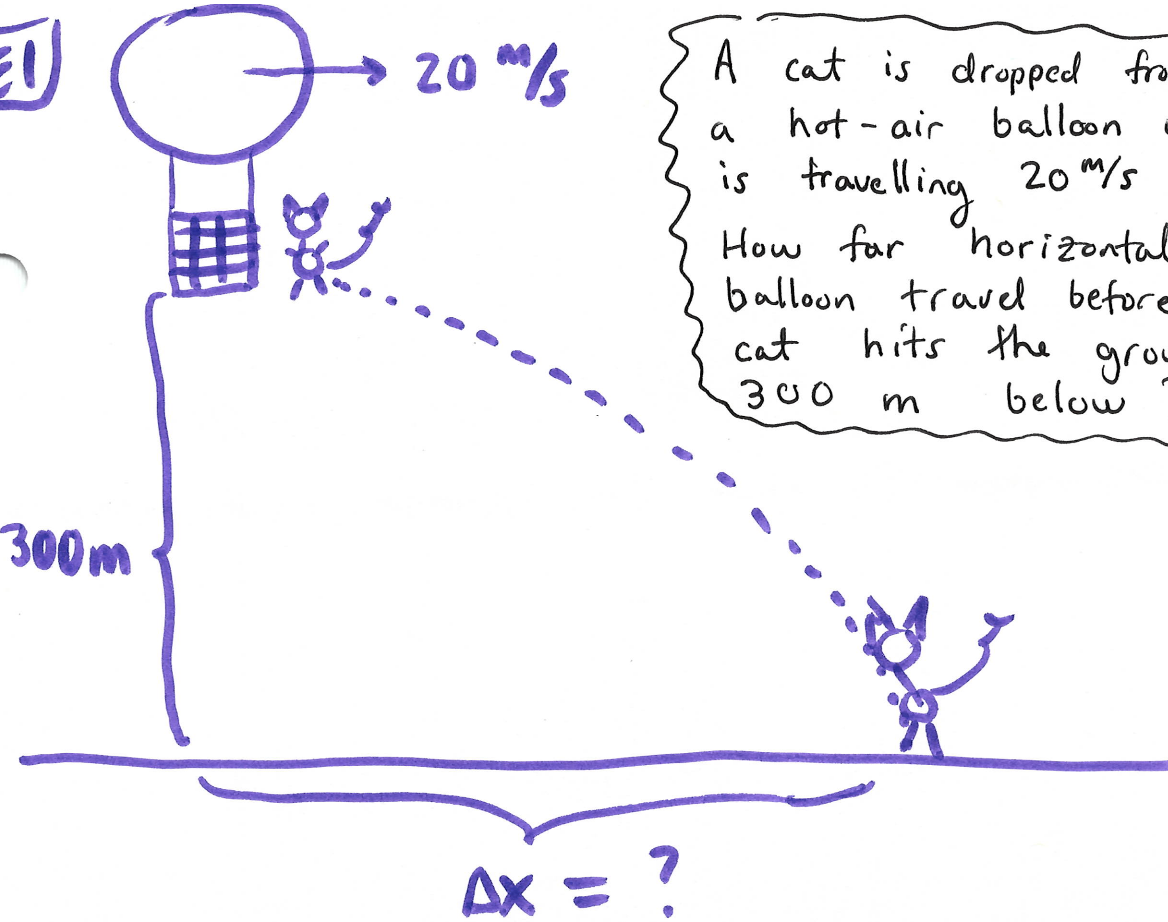
E1

A cat is dropped from a hot-air balloon which is travelling 20 m/s horizontally. How far horizontally does the balloon travel before the cat hits the ground some 300 m below?



Cat dropped, $\vec{V}_0 = \langle 20\ \text{m/s},\ 0 \rangle$

$$y = 300\text{m} - \frac{1}{2} g t^2$$

$$y(t_s) = 0 \Rightarrow t_s = \sqrt{\frac{2(300\text{m})}{9.8\ \text{m/s}^2}} = \underline{7.825\ \text{s}}.$$

$$\Delta x = (20\ \text{m/s})\, t_s = \boxed{156.5\ \text{m}}$$

E2

ASYMMETRIC RANGE PROBLEM

$$\Delta x = (V_0 \cos\theta)t$$

$$y = H + (V_0 \sin\theta)t - \frac{1}{2} g t^2 = 0$$

$$t^2 - \frac{2V_0 \sin\theta}{g} t - \frac{2H}{g} = 0$$

$$\left(t - \frac{V_0 \sin\theta}{g}\right)^2 - \frac{V_0^2 \sin^2\theta}{g^2} - \frac{2H}{g} = 0$$

$$\left(t - \frac{V_0 \sin\theta}{g}\right)^2 = \frac{V_0^2 \sin^2\theta + 2Hg}{g^2}$$

$$t = \frac{V_0 \sin\theta \pm \sqrt{V_0^2 \sin^2\theta + 2Hg}}{g} \qquad \text{choose } (+)$$

Remember $2\sin\theta\cos\theta = \sin(2\theta)$

$$\Delta x = \frac{1}{g}\left(V_0^2 \cos\theta \sin\theta + V_0 \cos\theta \sqrt{V_0^2 \sin^2\theta + 2Hg}\right)$$

$$= \frac{V_0^2}{2g} \sin(2\theta) + V_0^2 \sqrt{\cos^2\theta \sin^2\theta + \left(\frac{2Hg}{V_0^2}\right) \cos^2\theta}$$

$$= V_0^2 \left(\frac{\sin(2\theta)}{2g} + \sqrt{\frac{\sin^2(2\theta)}{4} + \frac{2Hg}{V_0^2} \cos^2\theta}\right)$$

$$\Delta X = \frac{1}{g}\left(V_0 \cos\theta \left[V_0 \sin\theta + \sqrt{V_0^2 \sin^2\theta + 2Hg} \right] \right)$$

$$R = \frac{V_0^2 \sin(2\theta)}{g}$$

Consider $V_0 = 1\ m/s$

$$H = \frac{6m}{9.8} \rightarrow 2Hg = 12m$$

$$R = \Delta X = \frac{1}{g}\left(\cos\theta \left[\sin\theta + \sqrt{\sin^2\theta + 12} \right] \right)$$

$$\frac{dR}{d\theta} = -\sin\theta \left[\sin\theta + \sqrt{\sin^2\theta + 12} \right] + \cos\theta \left[\cos\theta + \frac{\sin\theta\cos\theta}{\sqrt{\sin^2\theta + 12}} \right]$$

$$\frac{dR}{d\theta} = 0 = \frac{\left(-\sin^2\theta - \sin\theta\sqrt{\sin^2\theta + 12}\right)\sqrt{☺} + \cos^2\theta\sqrt{☺} + \sin\theta\cos\theta}{\sqrt{☺}}$$

I'm attempting to maximize R by finding its critical #'s Didn't work out too nice.

$$\Rightarrow -\sin^2\theta\sqrt{☺} - \sin\theta\left(\sin^2\theta + 12\right) + \cos^2\theta\sqrt{☺} + \sin\theta\cos\theta = 0$$

$$\left(\cos^2\theta - \sin^2\theta\right)\sqrt{\sin^2\theta + 12} = \sin^3\theta - 12\sin\theta - \sin\theta\cos\theta$$

$$\sqrt{\sin^2\theta + 12} = \frac{\sin^3\theta - 12\sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$\sin^2\theta + 12 = \left[\frac{\sin^3\theta - 12\sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}\right]^2$$

By Wolfram Alpha,

$$\theta \cong 0.27\,\text{rad} \cong \boxed{15.5^\circ}$$

If $H = \frac{12m}{9.8}$ then $\longrightarrow \theta \cong 0.197\,\text{rad} \cong \boxed{11.29^\circ}$

If $H = \frac{3m}{9.8}$ then $\longrightarrow \theta \cong 0.361\,\text{rad} \cong \boxed{20.7^\circ}$

If $H = \frac{1m}{9.8}$ then $\longrightarrow \theta \cong 0.524\,\text{rad} \cong \boxed{30.0^\circ}$

If $H = \frac{0.01m}{9.8}$ then $\longrightarrow \theta \cong 0.780\,\text{rad} \cong \boxed{44.7^\circ}$