$$B^2 = 2e^{i\frac{\pi}{2}} \rightarrow B = \sqrt{2}e^{i\frac{\pi}{4}}$$

$$z^2 - 2i = z^2 - 2e^{i\frac{\pi}{2}}$$

$$= (z-(1+i))(z+(1+i))$$

$$= \sqrt{2}(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}) = (\tfrac{\sqrt{2}}{2} + i\tfrac{\sqrt{2}}{2})\sqrt{2} = 1+i$$

$$z_1 = 1+i$$

$$z_2 = -1-i$$

$$z^2 + 2i = z^2 - 2(-i)$$

$$= z^2 - 2e^{i\frac{3\pi}{2}} \qquad B^2 = 2e^{i\frac{3\pi}{2}} \quad B = \sqrt{2}e^{i\frac{3\pi}{4}}$$

$$= (z - \sqrt{2}e^{\frac{3\pi i}{4}})(z + \sqrt{2}e^{\frac{3\pi i}{4}}) \qquad = \sqrt{2}(\cos\tfrac{3}{4}\pi + i\sin\tfrac{3}{4}\pi)$$

$$= (z-(-1+i))(z+(-1+i)) \qquad = \sqrt{2}(-1+i)$$

$$= (z+1-i)(z-1+i)$$

$$z_3 = -1+i \qquad z_4 = 1-i.$$

$$I = \oint_C \frac{dz}{z^4+4} = \left(\operatorname{Res}\left(\frac{1}{z^4+4}; z_1\right) + \operatorname{Res}\left(\frac{1}{z^4+4}; z_3\right)\right)2\pi i$$

$$= 2\pi i\left(\frac{1}{\underset{z_3, z_4}{(z^2+2i)}\underset{z_1}{(z-(1+i))}}\Bigg|_{\underset{z_1}{z=1+i}} + \frac{1}{\underset{z_1, z_2}{(z^2-2i)}(z-(-1+i))}\Bigg|_{\underset{z_3}{z=-1+i}}\right)$$

$$= \frac{\pi}{4}.$$

But $I = P.V.\int_{-\infty}^{\infty} \frac{dx}{x^4+4} + \lim_{R\to\infty}\int_{C_R} \frac{dz}{z^4+4}$

Consider for $z \in C_R \Rightarrow |z| = R$.

$$\left|\frac{1}{z^4+4}\right| \le \frac{1}{||z^4|-|4||} = \frac{1}{|R^4-4|} = \frac{1}{R^4-4}.$$

$$\lim_{R\to\infty}\int_{C_R} \frac{dz}{z^4+4} \le \lim_{R\to\infty}\frac{\pi R}{R^4-4} = 0.$$

$$\int_{C_R} \frac{P(z)}{Q(z)}dz = 0$$

if $\deg Q \ge \deg P + 2$

$$\operatorname*{Res}_{z=a}\left(\frac{\phi(z)}{z-a}\right) = \phi(a) = \phi^{\circ}(a).$$

ϕ entire, or at least analytic at $z=a$.

$$\operatorname{Res}\left(\frac{\phi(z)}{(z-a)^{n}}\right) = \phi^{(n-1)}(a).$$

even fnct. $\int_{-R}^{R} f(x)\,dx = 2\int_0^R f(x)dx.$

Theorem to. $\int_0^\infty f(x)\,dx = \frac{1}{2}\,\text{P.V.}\int_{-\infty}^{\infty} f(x)dx$

if $f(x) = f(x)$.

$\int \frac{\pm e^{3iz}dz}{z(z^4+4)}$

$$\int_{-\infty}^{\infty} \sin(mx)\frac{P(x)}{Q(x)}dx \quad \text{OR} \quad \int_{-\infty}^{\infty} \cos(mx)\frac{P(x)}{Q(x)}dx$$

we'll look at.

$$\underbrace{\int_{-\infty}^{\infty} e^{imx}\frac{P(x)}{Q(x)}dx}.$$

found form $\int_C e^{imz}\frac{P(z)}{Q(z)}dz$.

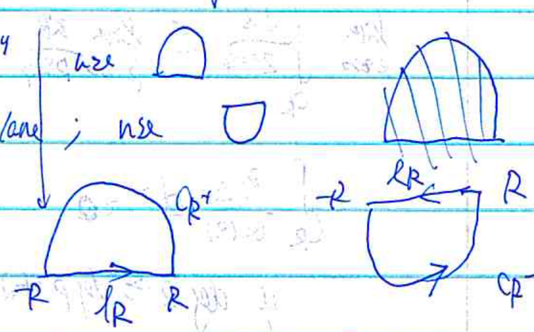
$\boxed{\text{Ex7}}$ P.V. $\int_{-\infty}^{\infty}\frac{\alpha\cos x}{x+i}dx \Leftarrow$ Jordan's Lemma gives us hope.

$$= \int \frac{\alpha\cos(z)}{z+i}dz = \int\frac{e^{iz}}{z+i}dz + \int\frac{e^{-iz}}{z+i}dz$$

bounded in upper half plane; Use ◠.

$\rightarrow e^{iz} = e^{i(x+iy)} = e^{ix-y} = e^{ix}e^{-y}$; use ◠

e^{-iz} bounded in lower half plane ; use ◡



$$\underbrace{\int_{C_R^+ \cup l_R} \frac{e^{iz}\,dz}{z+i}}_{0} = \int_{-R}^{R} \frac{e^{ix}\,dx}{x+i} + \underbrace{\int_{C_R^+} \frac{e^{iz}\,dz}{z+i}}_{0}$$ By Jordan's Lemma

Cauchy's Integral Thm.

$$\boxed{P.V. \int_{-\infty}^{\infty} \frac{e^{ix}\,dx}{x+i} = 0.}$$

Next.

$$\int_{C_R^- \cup l_R} \frac{e^{-iz}\,dz}{z+i} = \int_{-R}^{R} \frac{e^{-ix}\,dx}{x+i} + \int_{C_R^-} \frac{e^{-iz}\,dz}{z+i}$$ Jordan's lemma.

$$\xrightarrow{R\to\infty} \quad 2\pi i e^{-i(-i)} = P.V. \int_{-\infty}^{\infty} \frac{e^{-ix}}{x+i}\,dx + 0.$$

Thus,

$$P.V. \int_{-\infty}^{\infty} \frac{2\cos(x)\,dx}{x+i} = \frac{2\pi i}{e}$$

Curiousity:

$$\cancel{\frac{2\cos(x)}{x+i}\left(\frac{x+i}{x+i}\right)}$$

$$\frac{2\cos(x)}{x+i}\left(\frac{x-i}{x-i}\right) = \frac{2x\cos x}{x^2+1} - \frac{2i\cos x}{x^2+1}$$

$$P.V. \int_{-\infty}^{\infty} \frac{2x\cos(x)\,dx}{x^2+1} = 0.$$

$$P.V. \int_{-\infty}^{\infty} \frac{-2\cos x\,dx}{x^2+1} = \frac{2\pi}{e}.$$

$$\int_{0}^{\infty} \frac{\cos x\,dx}{x^2+1} = \frac{-\pi}{e}.$$

$$I = P.V. \int_{-\infty}^{\infty} \frac{e^{ix}dx}{x}$$

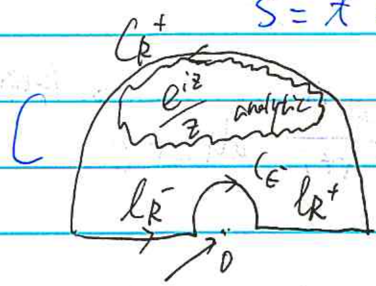
$$= \lim_{R\to\infty}\left(\int_{-R}^{0} \frac{e^{ix}dx}{x} + \int_{0}^{R} \frac{e^{ix}dx}{x}\right)$$

$$= \lim_{R\to\infty} \lim_{\epsilon\to 0^+} \left(\int_{-R}^{-\epsilon} \frac{e^{ix}dx}{x} + \int_{\epsilon}^{R} \frac{e^{ix}dx}{x}\right).$$

ϵ - connected goes with "P.V".

$$\int_{-1}^{1} \frac{dx}{x} = \lim_{t\to 0^-} \int_{-1}^{t} \frac{dx}{x} + \lim_{s\to 0^+} \int_{s}^{1} \frac{dx}{x}$$

$$s = t = \epsilon.$$

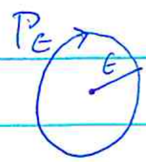


$$\int_{\substack{C \\ R\to\infty}} \frac{e^{iz}dz}{z} = 0 = \int_{l_R \cup l_{R^+}} \frac{e^{ix}dx}{x} + \int_{C_\epsilon^-} \frac{e^{iz}dz}{z}$$

$-\pi i.$

odd fct 0. even

problematic for $\frac{e^{iz}}{z}$.



$$\int_{P_\epsilon} \frac{e^{iz}}{z} dz = -2\pi i e^0 = -2\pi i.$$

$$\Rightarrow P.V. \int_{-\infty}^{\infty} \frac{e^{ix}dx}{x} = \pi i$$

$$= P.V. \int_{-\infty}^{\infty} \frac{i\sin(x)\,dx}{x} + P.V. \int_{-\infty}^{\infty}$$