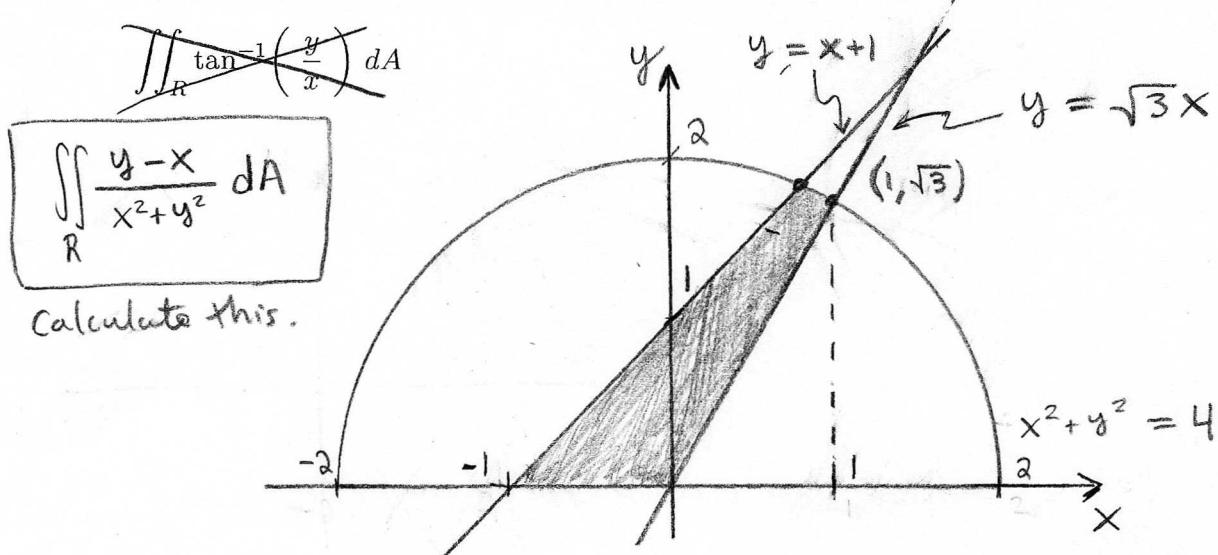


(1)

Calculus III, Bonus Take Home Work for Test 2, October 29-November 3, 2008 Name: _____

Credit will be awarded for correct content and clarity of presentation. This assignment can add 20pts to your Test 2 score. Partial credit is also awarded if need be. You may consult your text and my website and notes but no working with others. You should not even discuss level of difficulty. This is to be done individually. You are free to ask me questions, but I will probably not help unless it is an issue of lack of clarity in the statement of the question.

- 1) [20pts] Suppose R is the region bounded by $y = 1 + x$ and $y = \sqrt{3}x$ and $y = 0$ and the circle $x^2 + y^2 = 4$. Calculate the following integral by making a change of variables to polar coordinates. Graph the region R carefully before you begin the integration.



Notice $y = \sqrt{3}x$ intersects $x^2 + y^2 = 4$ where both eq's hold true $x^2 + 3x^2 = 4 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
 we are interested in $y \geq 0$ and by the picture we see
 $x = 1$ is the solⁿ which gives intersection point in quadrant I.
 Is my picture correct? Where does $y = x + 1$ intersect
 the circle?

$$x^2 + y^2 = 4 \quad \text{and} \quad y = x + 1 \quad \text{at intersection}$$

$$x^2 + (x+1)^2 = 4$$

$$x^2 + x^2 + 2x + 1 = 4$$

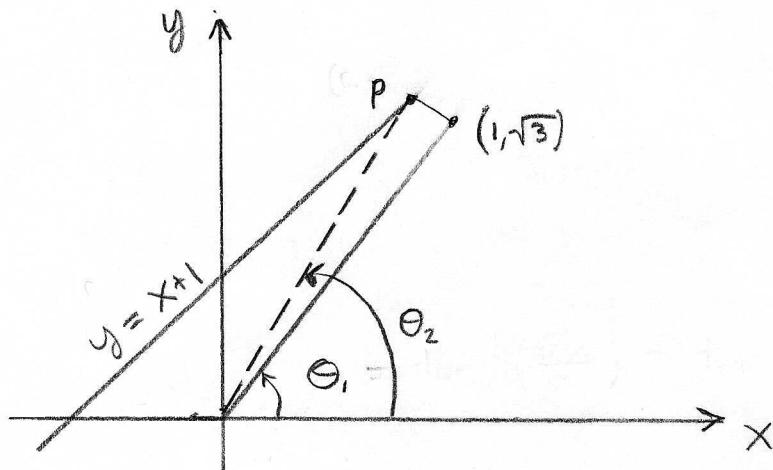
$$2x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 24}}{4} = \frac{-2 \pm 2\sqrt{7}}{4} = \frac{-1 \pm \sqrt{7}}{2}$$

The point of interest has $x = \frac{-1 + \sqrt{7}}{2} \approx 0.8229$.

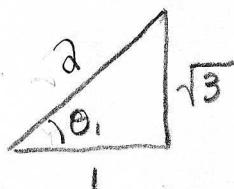
Sol^r to Bonus Problem Continued

(2)



Point P has $x \approx 0.8229$
and $y = x + 1$ so the
point $P \approx (0.8229, 1.8229)$

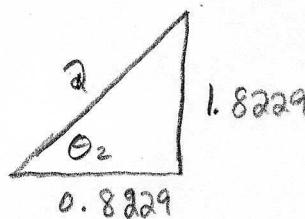
We can draw triangles to help figure θ_1 & θ_2



$$\theta_1 = \tan^{-1}(\sqrt{3})$$

$$\theta_1 = 1.0472 \text{ radians}$$

$$(\theta_1 = 60^\circ \text{ or } \frac{\pi}{3} \text{ rad.})$$



$$\theta_2 = \tan^{-1}\left(\frac{1.8229}{0.8229}\right)$$

$$\theta_2 = 1.1468 \text{ radians}$$

$$(\theta_2 = 65.7^\circ)$$

For later

$$\cos(\theta_1) = \frac{1}{2}$$

$$\sin(\theta_1) = \frac{\sqrt{3}}{2}$$

$$\sin(\theta_2) = \frac{1.8229}{2}$$

$$\cos(\theta_2) = \frac{0.8229}{2}$$

We have to break it into two cases.

I.) $|\frac{\pi}{3}| \leq \theta \leq 1.1468$ with $0 \leq r \leq 2$

II.) $1.1468 \leq \theta \leq \pi$ with $y \leq x + 1$
a.k.a $r \sin \theta \leq r \cos \theta + 1$
with

$$0 \leq r \leq \frac{1}{\sin \theta - \cos \theta}$$

$$r(\sin \theta - \cos \theta) \leq 1$$

$$r \leq \frac{1}{\sin \theta - \cos \theta}$$

makes

with a problem like this its a good idea to challenge your assertions. } sense, when $\theta = \pi$ get $r = \frac{1}{0-(0-1)} = 1$
when $\theta = \pi/2$ get $r = \frac{1}{1-0} = 1$.
when $\theta = 1.1468$ get $\cos(1.1468) = \frac{0.8229}{2}$
and $\sin(1.1468) = \frac{1.8229}{2}$ hence $r = \frac{1}{\frac{1.8229}{2} - \frac{0.8229}{2}} = 2$.

(3)

$$R_1 : \frac{\pi}{3} \leq \theta \leq 1.1468 \quad \text{with } 0 \leq r \leq 2$$

$$R_2 : 1.1468 \leq \theta \leq \pi \quad \text{with } 0 \leq r \leq \frac{1}{\sin\theta - \cos\theta}$$

$$\begin{aligned}
 \iint_{R_1} \frac{y-x}{x^2+y^2} dA &= \int_{\pi/3}^{1.1468} \int_0^2 \frac{(\sin\theta - \cos\theta)r}{r} dr d\theta \\
 &= \int_{1.1468}^{1.1468} (\sin\theta - \cos\theta) r \Big|_0^2 d\theta \\
 &= 2(-\cos\theta - \sin\theta) \Big|_{\pi/3}^{1.1468} \\
 &= 2(-\cos(1.1468) - \sin(1.1468) + \cos(\pi/3) + \sin(\pi/3)) \\
 &= 2\left(-\frac{0.8229}{2} - \frac{1.8229}{2} + \frac{1}{2} + \frac{2}{\sqrt{3}}\right) \\
 &= -0.8229 - 1.8229 + 2.7320 \\
 &\approx 0.08622
 \end{aligned}$$

$$\begin{aligned}
 \iint_{R_2} \frac{y-x}{x^2+y^2} dA &= \int_{1.1468}^{\pi} \int_0^{\frac{1}{\sin\theta - \cos\theta}} (\sin\theta - \cos\theta) dr d\theta \\
 &= \int_{1.1468}^{\pi} \left(\frac{\sin\theta - \cos\theta}{\sin\theta - \cos\theta} \right) d\theta \\
 &= \pi - 1.1468 \\
 &\approx 1.995
 \end{aligned}$$

$$\therefore \iint_R \frac{y-x}{x^2+y^2} dA = 0.08622 + 1.995 \approx \boxed{2.081}$$