MATRIX ENCRYPTION

First, let's agree on the following basic rules:

0 = Space		13 = M	
1 = A		14 = N	
2 = B		15 = O	
3 = C		16 = P	
4 = D		17 = Q	
5 = E		18 = R	
6 = F		19 = S	
7 = G		20 = T	
8 = H		21 = U	
9 = I		22 = V	
10 = J		23 = W	
11 = K		24 = X	
12 = L		25 = Y	
		26 = Z	

$\boxed{\text{E249}}$ Code the phrase using three component row vectors.

HAVE A NICE DAY

$[8, 1, 22][5, 0, 1][0, 14, 9][3, 0, 4][1, 25, 0]$

$\boxed{\text{E250}}$ Code the phrase "Mommy" using 2-component row vectors

$[13, 15][13, 13][25, 0]$

$\boxed{E251}$ Use $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ to encrypt Mommy

$$[13, 15]\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = [15, 43]$$

$$[13, 13]\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = [13, 39]$$

$$[25, 0]\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = [0, 25]$$

Now the phrase "Mommy" is encrypted into $[15, 43], [13, 39], [0, 25]$.
Notice we can unscramble the string using the inverse matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{0-1}\begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$$

Then multiply on the right,

$$[15, 43]\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} = [13, 15]$$

$$[13, 39]\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} = [-26+39, 13] = [13, 13]$$

$$[0, 25]\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} = [25, 0]$$

And we recover the string $[13, 15][13, 13][25, 0]$ which translates into M O M M Y via our conventional alphabet code on 132.

$\boxed{E252}$ Suppose the following string is encrypted as in $\boxed{E251}$. What is the hidden phrase?

$$[15, 37][0, 4][19, 47][15, 37][4, 23],$$

__ __ __ __ __ __ __ __ __ __,

Fill in the blanks.

Our Encryption System

1.) Convert given phrase into string of vectors using the standard code on pg. 132

2.) Choose an encryption matrix A with $\det(A) \neq 0$ since we need A^{-1} to exist for decryption.

3.) Multiply each vector in the string of vectors by A on the right to obtain the encrypted message.

4.) Decrypt the message by multiplying by A^{-1} on the right. Then again use code on pg. 132 to convert back to letters and spaces.

$\boxed{\text{E253}}$ Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ was use the encode a message into the string below,

$[1, 12, 12], [0, 20, 8], [5, 0, 20], [9, 13, 5]$

__ __ __   __ __ __   __ __ __   __ __ __ !

Fill in the blanks. (Note $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.)

$\boxed{\text{E254}}$ Decode the following message. (same encryption matrix as $\boxed{\text{E253}}$)

$[20, 28, 33]\,[0, 5, 19]\,[4\ 4\ 4]$

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