HOMOGENEOUS LINEAR EQ^{s}'s : § 4.2

§ 4.2 # 1

$$2y'' + 7y' - 4y = 0$$

$$2\lambda^2 + 7\lambda - 4 = 0$$

$$(2\lambda - 1)(\lambda + 4) = 0 \quad \therefore \lambda_1 = 1/2, \ \lambda_2 = -4$$

$$\Rightarrow \boxed{y = c_1 e^{x/2} + c_2 e^{-4x}}$$

§ 4.2 # 5

$$y'' + 8y' + 16y = 0$$

$$\lambda^2 + 8\lambda + 16 = 0$$

$$(\lambda + 4)^2 = 0 \Rightarrow \lambda_1 = \lambda_2 = -4 \Rightarrow \boxed{y = c_1 e^{-4t} + c_2 t e^{-4t}}$$

§ 4.2 # 13

$$y'' + 2y' - 8y = 0, \quad y(0) = 3 \ \& \ y'(0) = -12$$

$$\lambda^2 + 2\lambda - 8 = 0$$

$$(\lambda + 4)(\lambda - 2) = 0$$

$\lambda_1 = -4, \lambda_2 = 2$ hence $y = c_1 e^{-4t} + c_2 e^{2t}$

$$y' = -4c_1 e^{-4t} + 2c_2 e^{2t}$$

We can specify c_1 & c_2 via the initial conditions,

$$y(0) = c_1 + c_2 = 3 \Rightarrow \begin{pmatrix} 2c_1 + 2c_2 = 6 \\ -4c_1 + 2c_2 = -12 \end{pmatrix}$$

$$y'(0) = -4c_1 + 2c_2 = -12 \quad -$$

$$6c_1 = 18 \quad \therefore \underline{c_1 = 3}$$

$$c_2 = 3 - c_1 = 3 - 3 = 0.$$

Therefore, $\boxed{y = 3e^{-4t}}$

§4.2#17)

$$z'' - 2z' - 2z = 0, \quad z(0) = 0 \;\&\; z'(0) = 3.$$

$$\lambda^2 - 2\lambda - 2 = 0$$

$$\lambda = \frac{2 \pm \sqrt{4-4(-2)}}{2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

$$z = c_1 e^{(1+\sqrt{3})t} + c_2 e^{(1-\sqrt{3})t}$$

$$z' = c_1(1+\sqrt{3})e^{(1+\sqrt{3})t} + c_2(1-\sqrt{3})e^{(1-\sqrt{3})t}$$

$$z(0) = c_1 + c_2 = 0$$

$$z'(0) = \underbrace{c_1(1+\sqrt{3}) + c_2(1-\sqrt{3}) = 3}$$

$$c_1(1+\sqrt{3}) - c_1(1-\sqrt{3}) = 3$$

$$2\sqrt{3}\, c_1 = 3 \longrightarrow \underline{c_1 = \frac{\sqrt{3}}{2} \text{ and } c_2 = \frac{-\sqrt{3}}{2}}$$

Thus, $\boxed{z = \frac{\sqrt{3}}{2} e^{(1+\sqrt{3})t} - \frac{\sqrt{3}}{2} e^{(1-\sqrt{3})t}}$

Could write $z = \sqrt{3}\, e^{t} \sinh(\sqrt{3}\, t)$.

§4.2#27) Use def$^{\underline{n}}$ to determine if $y_1(t) = \cos t \sin t$ and $y_2(t) = \sin(2t)$ are linearly dependent on $(0,1)$

Notice $\sin(2t) = 2\cos t \sin t$ thus $y_2(t) = 2y_1(t) \;\; \forall t \in (0,1)$.
Therefore y_1 and y_2 are <u>linearly</u> <u>dependent</u> on $(0,1)$.

§4.2#29) Are $y_1(t) = te^{2t}$ and $y_2(t) = e^{2t}$ linearly dependent on $(0,1)$?

Proceed by contradiction. Suppose $\exists$ constant k such that $y_1(t) = k\, y_2(t) \;\; \forall t \in (0,1)$. Then

$$te^{2t} = ke^{2t} \implies t = k$$

which clearly contradicts $\frac{dk}{dt} = 0$ (k is constant iff $\frac{dk}{dt} = 0$).
Thus y_1 and y_2 are <u>linearly</u> <u>independent</u> on $(0,1)$.