THE ALGEBRA I SKIPPED 1/20/2010

$$A_1 C_1 + A_2 C_2 + A_3 C_3 = 0 \qquad A \cdot C = 0$$
$$B_1 C_1 + B_2 C_2 + B_3 C_3 = 0 \qquad B \cdot C = 0$$

$$\therefore \; C_1 = \frac{-A_2 C_2 - A_3 C_3}{A_1} \quad \& \quad C_1 = \frac{-B_2 C_2 - B_3 C_3}{B_1}$$

$$\frac{-A_2 C_2 - A_3 C_3}{A_1} = \frac{-B_2 C_2 - B_3 C_3}{B_1}$$

$$\frac{A_2 C_2}{A_1} + \frac{A_3 C_3}{A_1} = \frac{B_2 C_2}{B_1} + \frac{B_3 C_3}{B_1}$$

$$C_2 \left(\frac{A_2}{A_1} - \frac{B_2}{B_1} \right) = C_3 \left(\frac{B_3}{B_1} - \frac{A_3}{A_1} \right)$$

$$C_2 = \frac{1}{\frac{A_2}{A_1} - \frac{B_2}{B_1}} \left[\frac{B_3}{B_1} - \frac{A_3}{A_1} \right] C_3$$

$$= \frac{1}{B_1 A_2 - A_1 B_2} \left[A_1 B_3 - B_1 A_3 \right] C_3$$

$$\underline{\underline{C_2 = \left(\frac{A_1 B_3 - B_1 A_3}{B_1 A_2 - A_1 B_2} \right) C_3}}$$

Then we can also write C_1 with this

$$C_1 = -\frac{B_2}{B_1} C_2 - \frac{B_3}{B_1} C_3$$

$$= \left[-\frac{B_2}{B_1} \left(\frac{A_1 B_3 - B_1 A_3}{B_1 A_2 - A_1 B_2} \right) - \frac{B_3}{B_1} \right] C_3$$

$$= \left[\frac{-A_1 B_2 B_3 + B_2 B_1 A_3 - B_3 (B_1 A_2 - A_1 B_2)}{B_1 (B_1 A_2 - A_1 B_2)} \right] C_3$$

$$= \left[\frac{\cancel{-A_1 B_2 B_3} + B_2 \cancel{B_1} A_3 - B_3 \cancel{B_1} A_2 + \cancel{B_3 A_1 B_2}}{\cancel{B_1} (B_1 A_2 - A_1 B_2)} \right] C_3$$

$$= \left[\frac{B_2 A_3 - B_3 A_2}{B_1 A_2 - A_1 B_2} \right] C_3$$

Remark: this derivation assumes we can divide by A_1, A_2, B_1, B_2 etc... but generally this is false. To give complete argument need to consider cases

$$\therefore \; \boxed{\vec{C} = \frac{-C_3}{B_1 A_2 - A_1 B_2} \left\langle A_2 B_3 - B_2 A_3 , \; A_3 B_1 - A_1 B_3 , \; A_1 B_2 - B_2 A_1 \right\rangle}$$

choose $C_3 = A_1 B_2 - A_2 B_1$ and get $\vec{C} = \vec{A} \times \vec{B}$.