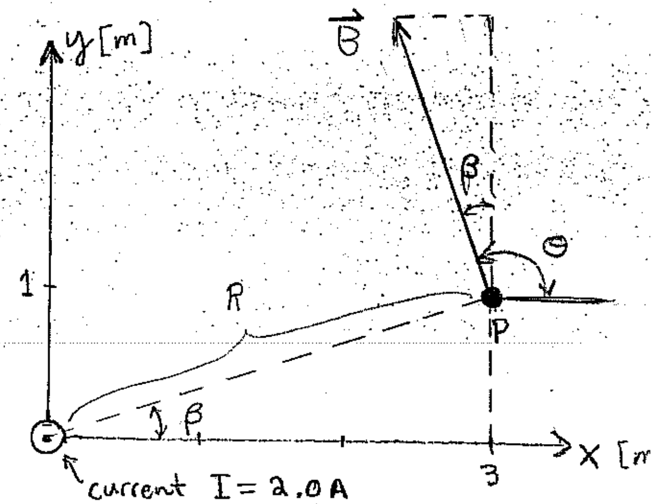
Do not omit scratch work. I need to see all steps. Skipping details will result in a loss of credit. Thanks. Note the term "find" in this test does not merely mean to guess the answer. Without exception the term "find" means to derive from proper physical and mathematical arguments the desired item.

In this test the symbol ⊗ indicates a vector going directly into the page. To indicate a vector coming out of the page I will use the symbol ⊙.

Problem 1 [7pts] Suppose a very long wire carries a current of $I = 2.0\,A$ out of the page. Place the origin of your coordinate system at the wire. Find the magnetic field due to this current at the point $P = (3, 1)m$ as pictured. Calculate both the magnitude and direction (in terms of the standard angle) of the magnetic field at P.



$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7})(2.0)\,T}{2\pi\sqrt{3^2 + 1^2}}$$

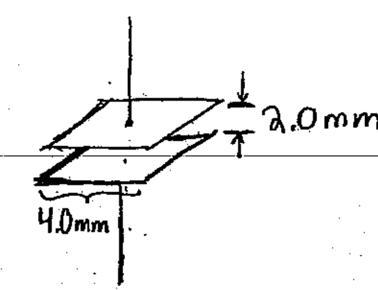
$$\boxed{B = 1.265 \times 10^{-7}\,T}$$

$$\tan\beta = \frac{1}{3} \rightarrow \beta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$$

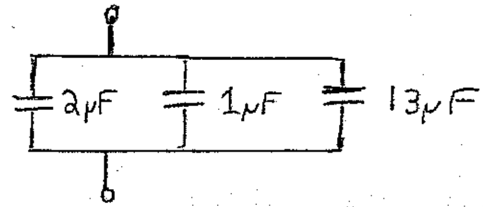
$$\boxed{\theta = 108.43^\circ}$$

Problem 2 [7pts] What is the capacitance of a parallel plate capacitor with square plates of side length $4.0\,mm$, a separation distance of $d = 2.0\,mm$ if the space between the plates is filled with air.



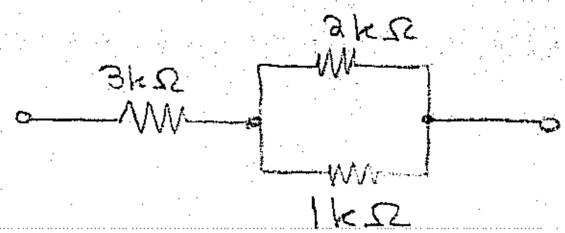
$$C = \frac{A\epsilon_0}{d} = \frac{(0.004)^2(8.85 \times 10^{-12})\,F}{0.002} = \boxed{7.08 \times 10^{-13}\,F}$$

Problem 3 [7pts] Find the equivalent capacitance of the network of capacitors.



$$C_{eq} = 2\mu F + 1\mu F + 13\mu F = \boxed{16\mu F}$$

Problem 4 [7pts] Find the equivalent resistance of the network of resistored.



Let $R = 1k\Omega$:

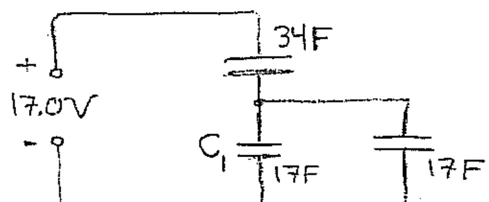
$$R_{eq} = 3R + \frac{1}{\frac{1}{2R} + \frac{1}{R}}$$

$$= 3R + \frac{1}{\frac{3}{2R}}$$

$$= 3R + \frac{2}{3}R$$

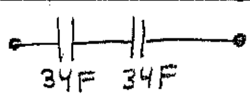
$$= \left(\frac{9+2}{3}\right)R = \boxed{\frac{11}{3}k\Omega \mathrel{\hat{=}} 3.667\,k\Omega}$$

Problem 5 [7pts] What is the charge Q_1 on C_1 if the pictured network of capcacitors is charged to a voltage of $17.0V$?



Notice $17F \parallel 17F \approx 34F$

$\therefore$ circuit $\approx$ series combo



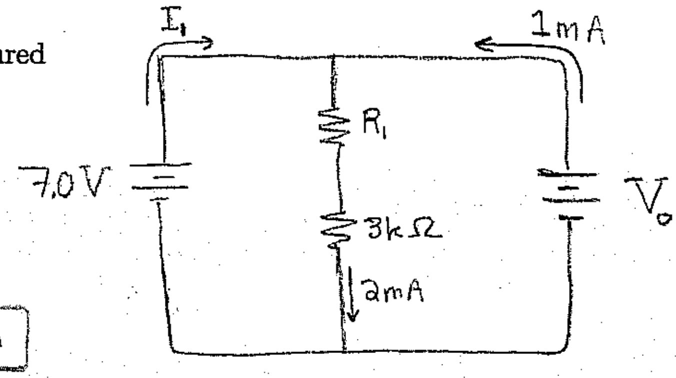
Thus $V_1 = \frac{17.0}{2}V$ by symmetry. $C_1 = \frac{Q_1}{V_1}$

$$\Rightarrow \quad Q_1 = C_1 V_1 = (17F)\left(\frac{17.0V}{2}\right)$$

$$\Rightarrow \quad \boxed{Q_1 = 144.5\,C}$$

Problem 6 [7pts] Calculate the following items for the pictured circuit:

- (a.) Current I_1
- (b.) Resistance R_1
- (c.) Voltage V_o



(a.) $I_1 + 1\text{mA} = 2\text{mA} \Rightarrow \boxed{I_1 = 1\text{mA}}$

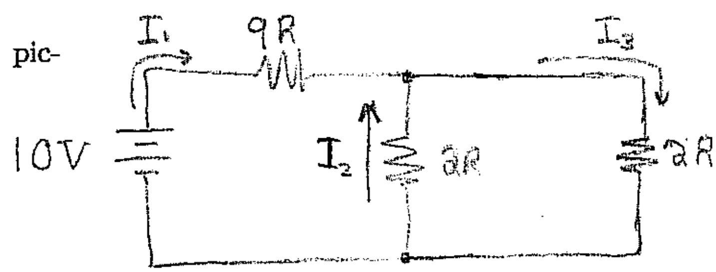
(b.) $(2\text{mA})R_1 + (2\text{mA})(3\text{k}\Omega) = 7.0\text{V}$

$\Rightarrow R_1 = \left(\frac{7.0 - 6.0}{0.002}\right)\Omega = \boxed{500\Omega = R_1}$

(c.) Since 7.0V is $\parallel$ with V_o $\Rightarrow \boxed{V_o = 7.0\text{V}}$

Problem 7 [12pts] Calculate the following items for the pictured circuit:

- (a.) Current I_1
- (b.) Current I_2
- (c.) Current I_3



$R = 10\text{k}\Omega$

$$\left.\begin{aligned} 10 - 9RI_1 + 2RI_2 &= 0 \\ -2RI_2 - 2RI_3 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned}\right\} \text{Solve.}$$

or, use equivalent resistance technique, $R_{eq} = 9R + \frac{1}{\frac{1}{2R} + \frac{1}{2R}}$

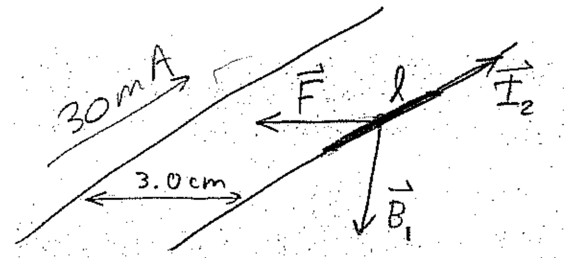
Thus $R_{eq} = 10R \Rightarrow I_1 = \frac{10\text{V}}{10R} = \boxed{0.1\text{mA} = I_1}$

Then, by symmetry, $I_3 = -I_2 \quad \therefore \quad I_1 = 2I_3$

We find $\boxed{I_2 = -0.05\text{mA}}$ & $\boxed{I_3 = 0.05\text{mA}}$

Problem 8 [7pts] Two parallel wires are separated by a distance of 3.0 cm are attracted towards each other via the magnetic force. In particular, it is measured that for every 1.0 m of wire there is an force of 0.025 N. If the current through one of the wires is 30.0 mA then

- (a.) find the current through the other wire
- (b.) does the second current flow in the same or in the opposite direction as the measured current?



(a.) $\dfrac{F}{l} = I_2 B_1 = \dfrac{\mu_0 I_1 I_2}{2\pi R}$

$\Rightarrow I_1 = \left(\dfrac{2\pi R}{\mu_0 I_2}\right)\left(\dfrac{F}{l}\right)$

$\Rightarrow I_1 = \left[\dfrac{2\pi(0.03)(0.025)}{(4\pi\times10^{-7})(0.030)}\right] A$

$\Rightarrow \boxed{I_1 = 125{,}000\ A}$

$F = l I_2 B_1$ — force on $\vec{I_2}$ over length l due to B_1 generated from $I_1 = 30mA$.

$B_1 = \dfrac{\mu_0 I_1}{2\pi R}$ since long & straight wire.

(b.) need same direction for attraction. (see picture)

Problem 10 [9pts] If $\vec{E} = E_o(\hat{\imath}+\hat{\jmath})$ and an unknown magnetic field $\vec{B}$ produces a force of $\vec{F_o} = qE_o(\hat{\imath}+\hat{\jmath}+\hat{k})$ on a mass $m = 1.0\,kg$ with charge q with velocity $\vec{v} = v_o(\hat{\imath}-\hat{\jmath})$ then **find all possible $\vec{B}$ which produce this outcome.**

let $\vec{B} = \langle a, b, c\rangle = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$\vec{F_o} = q\vec{E_o} + q\vec{v}\times\vec{B}$

$qE_o(\hat{\imath}+\hat{\jmath}+\hat{k}) = qE_o(\hat{\imath}+\hat{\jmath}) + qV_o(\hat{\imath}-\hat{\jmath})\times\vec{B}$

$\Rightarrow E_o\hat{k} = V_o(\hat{\imath}-\hat{\jmath})\times(a\hat{\imath}+b\hat{\jmath}+c\hat{k})$

$= V_o b\hat{k} - V_o c\hat{\jmath} + V_o a\hat{k} - V_o c\hat{\imath}$

$= V_o(b+a)\hat{k} - (V_o c)\hat{\jmath} - (V_o c)\hat{\imath}$

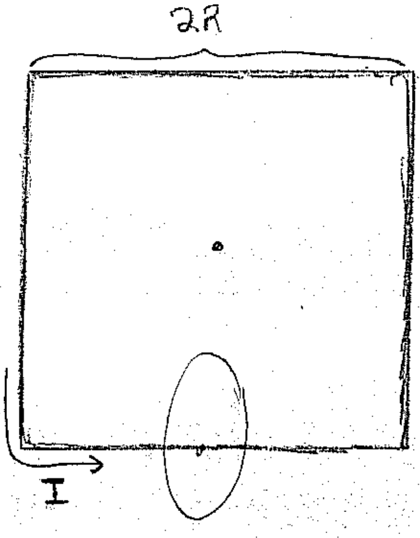
Thus, comparing coefficients of $\hat{\imath}, \hat{\jmath}, \hat{k}$, suppose $V_o \neq 0$

$E_o = V_o(a+b) \Rightarrow a+b = E_o/V_o \therefore \boxed{b = \dfrac{E_o}{V_o} - a}$

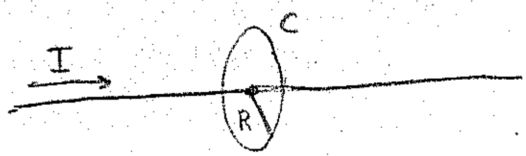
$0 = -V_o c$
$0 = -V_o c$ $\}\longrightarrow \boxed{c = 0}$

$\boxed{\vec{B} = a\hat{\imath} + \left(\dfrac{E_o}{V} - a\right)\hat{\jmath}}$ (for any $a \in \mathbb{R}$).

Problem 11 [7pts] Find the magnitude of the magnetic field generated at the center of a square loop of current I around a square with side-length $2R$. Derive the necessary magnetic field formula by applying Ampere's Law to an aptly chosen loop (you can do it once then re-use the result).



apply ampere's law to center of a side, use symmetry to simplify

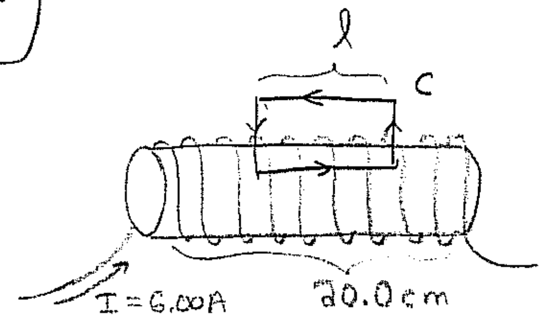


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi R) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

from just one side

4 sides $\Rightarrow$ $$B_{TOTAL} = \frac{2\mu_0 I}{\pi R}$$

Problem 12 [7pts] A solenoid of length 20.0 cm with N turns has a current of $I = 6.00\,A$ flowing. If the magnetic field strength near the center of the solenoid is measured to have a magnitude of $0.0188\,T$ then what is N ? Assume edge-effects are neglible.



$n = \frac{N}{L}$

$N = ?$ & $L = 20.0\,cm$

$$\int_C \vec{B}\cdot d\vec{l} = \mu_0 I_c = \mu_0 n l I$$

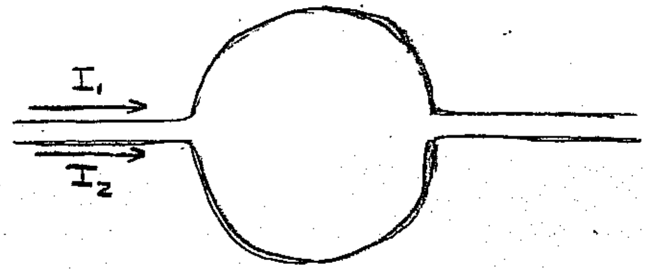
other 3 sides contribute nothing because $\perp$ or zero for top leg.

$B l = \mu_0 n l I \quad \therefore \quad B = \mu_0 n I$

$$B = \frac{\mu_0 N I}{L}$$

$$N = \frac{BL}{\mu_0 I} = \frac{(0.0188)(0.200)}{(4\pi\times10^{-7})(6.00)} = 498.7$$

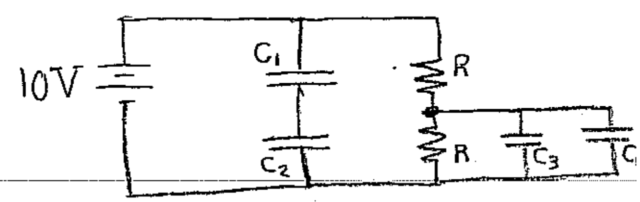
Problem 13 [7pts] A current of I_1 flows over the upper half of a circle of radius R and a current of I_2 flows over the lower half of the same circle. Both currents flow from left to right. What is the magnetic field at the center of the circle? Apply Biot-Savart law and explicitly set-up the integration which yields the magnetic field.



see PROBLEM 18 sol$^{\underline{n}}$

(from hwk.)

Problem 9 [7pts] Find the voltage and charge on each of the capacitors in the pictured circuit.



$C_1 = 2\mu F$ $\quad C_3 = 1nF$

$C_2 = 2\mu F$ $\quad C_4 = 3nF$

$C_1 = C_2 \Rightarrow$ 5V on C_1 & C_2

$R = R \Rightarrow$ 5V on C_3 & C_4

$Q_1 = (2\mu F)(5V) = 10\mu C = Q_1$

$Q_2 = (2\mu F)(5V) = 10\mu C = Q_2$

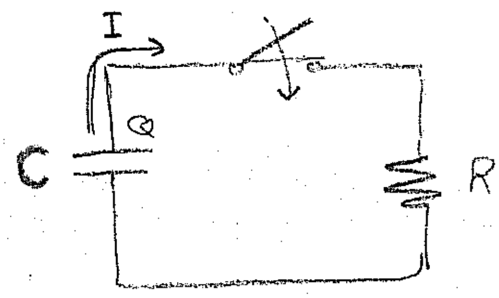
$Q_3 = (1nF)(5V) = 5nC = Q_3$

$Q_4 = (3nF)(5V) = 15nC = Q_4$

$C_1 = \frac{Q_1}{V}$

Problem 14 [12pts] Assume that the capacitor $C = 1.0\mu F$ is initially charged to a voltage of $3.0\,V$. At time $t = 0$ (in seconds) the switch is closed and the capacitor discharges through the resistor $R = 1.0\,M\Omega$.

- (a.) [8pts] Find the current I as a function of time for $t > 0$
- (b.) [2pts] Find the instantaneous power dissapated in the resistor at time $t > 0$
- (c.) [2pts] Find the total energy disapated in the resistor as $t \to \infty$



$$-\frac{dQ}{dt} = I$$

$$\frac{Q}{C} - IR = 0$$

$$\frac{dQ}{dt} = \frac{-Q}{RC} \longrightarrow \int \frac{dQ}{Q} = \int \frac{-dt}{RC}$$

$$\ln|Q| = \frac{-t}{RC} + C_1$$

$$Q(t) = e^{-t/RC} + C_1$$

$$Q(0) = V(0)C = (3.0V)(1.0\mu F) = 3\mu C. = e^{C_1} = Q_0$$

Thus $Q(t) = Q_0 e^{-t/\tau}$ where $\tau = RC$ & $Q_0 = 3\mu C$

(a.) $I(t) = -\frac{dQ}{dt} = -\frac{d}{dt}\left(Q_0 e^{-t/\tau}\right) = \frac{Q_0}{\tau} e^{-t/\tau}$

Note $RC = \tau = 1\,s$ thus $\boxed{I(t) = (3.0\mu C)e^{-t}}$

$(p = 10^{-12})$

(b.) $P(t) = I^2(t)R = (9.0\mu^2)(1)(e^{-t})^2 = \boxed{9.0\,e^{-2t}\,pW}$

(c.) $U_C = \frac{1}{2}CV^2 = \frac{1}{2}(1.0\mu F)(3.0V)^2 = \boxed{4.5\mu J}$

or, since $P = \frac{dE}{dt} = (9\times10^{-12})e^{-2t} \Rightarrow dE = (9\times10^{-12})e^{-2t}\,dt$

$$E = (9\times10^{-12})e^{-2t}\Big|_0^\infty$$

$$\Rightarrow \boxed{E = 4.5\times10^{-12}\,J}$$