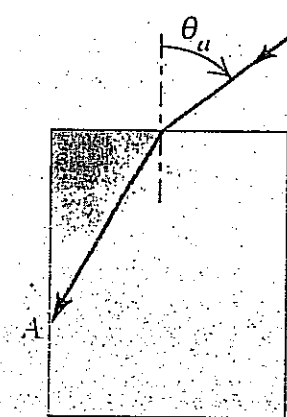
Do not omit scratch work. I need to see all steps.

1. The current in a 0.400 H inductor is given by $I = at^3 - bt^2$ where I is in amperes and t is in seconds. The numerical values of the constants a and b are 2.00 and 5.00, respectively. At time t = 3.00 s, find (a) the potential difference across the inductor and (b) the energy stored in the inductor.

2. A ray of light is incident on a block of ice (n = 1.31). If the angle of incidence is 60°, find the angle of refraction. (b) Will the ray be totally internally reflected when it reaches point A? How do you know?



Physics 1151 Fourth 50-minute Examination Spring, 1998

8 3% AVG.

Name JAMES COOK

Please answer all five of the following. Partial credit may be given. However, credit cannot be awarded unless sufficient work is shown.

1. The current in a 0.400 H inductor is given by $I = at^3 - bt^2$ where I is in amperes and t is in seconds. The numerical values of the constants a and b are 2.00 and 5.00, respectively. At time t = 3.00 s, find (a) the potential difference across the inductor and (b) the energy stored in the inductor.

+20

a) $\mathcal{E}_2 = L\frac{dI}{dt}$ $L = 400mH$

$I = 2t^3 - 5t^2 = 9A$

$\frac{dI}{dt} = \frac{d}{dt}(I) = \frac{d}{dt}[2t^3 - 5t^2]$

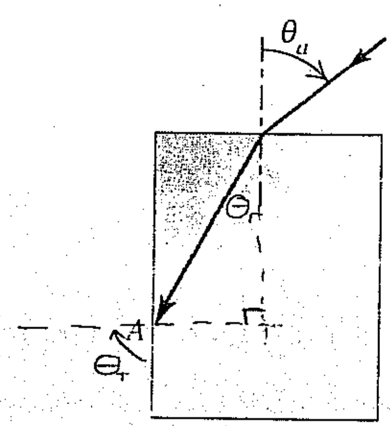
$\frac{dI}{dt} = 6t^2 - 10t$

$\mathcal{E}_2 = L(6t^2 - 10t) = \boxed{9.6V = \mathcal{E}_2}$

b) = $U = \frac{1}{2}LI^2 = \boxed{16.2 J = U}$

2. A ray of light is incident on a block of ice (n = 1.31). If the angle of incidence is 60°, find the angle of refraction. (b) Will the ray be totally internally reflected when it reaches point A? How do you know?

+20



a) (1) $\sin 60° = 1.31 \sin\theta_r$

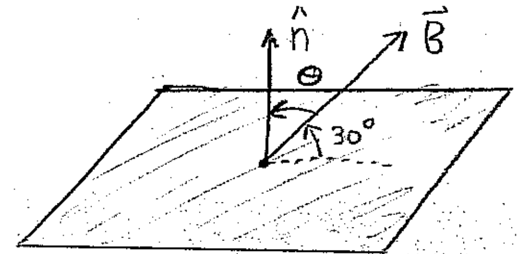
$\theta_r = \sin^{-1}\left[\frac{\sin 60}{1.31}\right] = \boxed{41.38° = \theta_r}$

B) $\sin\theta_T = (\cos\theta_r)1.31$

if $\theta_T = 90° \rightarrow 0°$ THEN TOTAL INTERNAL REFLECTION DOESN'T HAPPEN

$\theta_T = \sin^{-1}\left[\cos\theta_r \; 1.31\right] = 79.4°$, so NO THE RAY IS PARTIALLY REFRACTED SO SOME GETS AWAY.

Problem 4 Calculate the magnetic flux through the surface S due to the constant magnetic field $\vec{B}$.



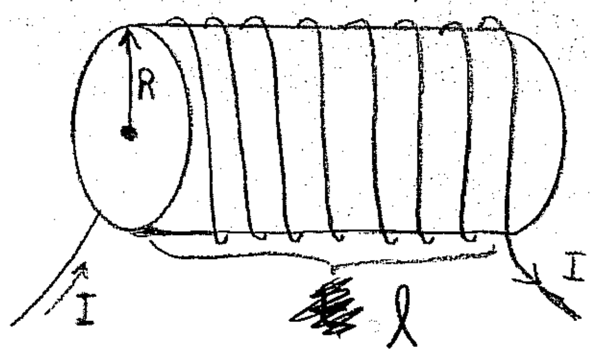
$\hat{n}$ is normal to square S with side-length L.

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A}$$
$$= \vec{B} \cdot \vec{A}$$
$$= BA\cos\theta \quad , \cos 60° = \frac{1}{2}$$
$$= \boxed{\frac{BL^2}{2}}$$

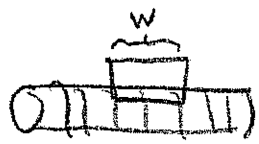
$\ell = 50cm$

Problem 5 Suppose a cylindrical solenoid has 1000 turns per meter and a total length of ~~[illegible]~~. Suppose that the radius of the solenoid is $R = 10.0\,mm$. What is the self-inductance of this solenoid? If a current $I_o = 25.0\mu A$ flows through the solenoid then how much energy is stored in the magnetic field of the coil. (please ignore end-effects, you may assume this is a "very long" solenoid for the purposes of your calculations)

(not to scale)



By Ampere's Law



$$n = \frac{N}{\ell}$$

$$\mu_o I n w = B w$$
$$\Rightarrow \underline{B = \mu_o n I}$$

$$\Phi_B = LI = NBA = (n\ell)(\mu_o n I)(\pi R^2)$$

$$L = \mu_o \pi n^2 \ell R^2$$
$$= (4\pi^2 \times 10^{-7})(1000)^2 (0.5)(0.010)^2 \ H$$
$$= \boxed{197\mu H = L}$$

$$U = \frac{1}{2} L I^2$$
$$= \frac{1}{2}(197 \times 10^{-6})(25 \times 10^{-6})^2 \ J$$
$$= \boxed{6.17 \times 10^{-14}\ J}$$

Problem 6 State Maxwell's Equations.

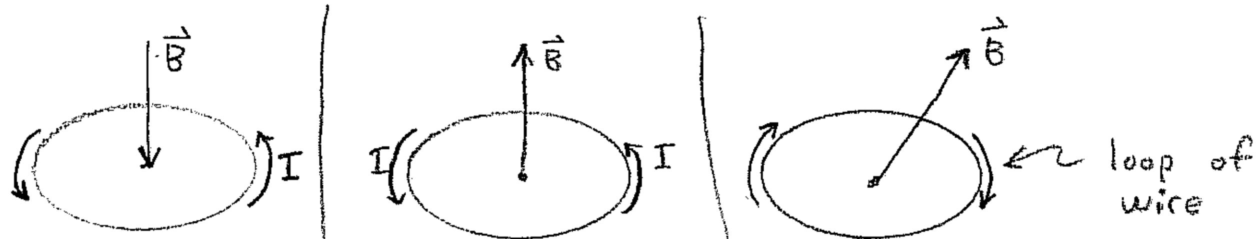
$$d(*F) = \mu_0 (*J)$$
$$dF = 0.$$

Problem 7 An inductor stores $10.0\mu J$ of energy when a current of $I = 2.0mA$ is flowing through its coils. Find the inductance.

$$U = \frac{1}{2} L I^2$$

$$L = \frac{2U}{I^2} = \frac{2(10.0\mu J)}{(2.0mA)^2} = \boxed{5\,H = L}$$

Problem 8 Indicate the direction of the induced current for each case illustrated. In each case $|\vec{B}| = g(t)$



$\frac{dg}{dt} > 0$

flux increasing downward $\Rightarrow$ induced current CCW (see picture)

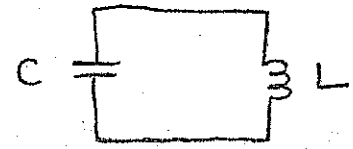
$\frac{dg}{dt} < 0$

Same, flux downward increasing

$\frac{dg}{dt} > 0$

flux upward increasing $\Rightarrow$ CW induced current

Problem 9 An LC-circuit is observed to have a current with frequency $f = 10.0\,MHz$. If $C = 2.0\mu F$ then what is the value of L? How much time does it take the circuit to go from zero current to maxium current?



$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$4\pi^2 f^2 = \frac{1}{LC}$$

$$\therefore\ L = \frac{1}{4\pi^2 f^2 C} = \boxed{1.267\times10^{-10}\,H}$$

$$f = \frac{1}{T} = 10^7\,\frac{1}{s}$$

$$\Rightarrow T = 10^{-7}\,s$$

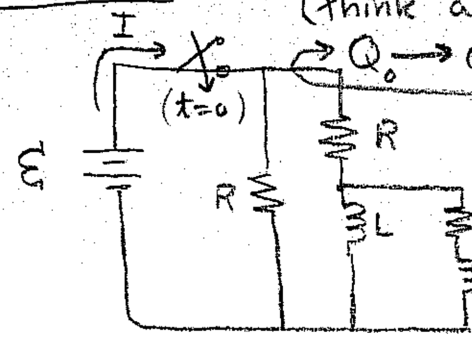
$$\Rightarrow \boxed{t = 0.5\times10^{-7}\,s}$$

(think about it, $Q_0 \rightarrow 0 \rightarrow -Q_0$
0
two zeros per cycle

Problem 10 For the circuit pictured, find:

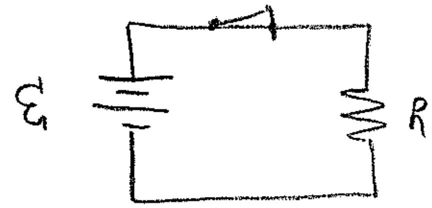
- (a.) I for a very small time; $t = 0^+$
- (b.) I for a very large time; $t >> 0$

(switch closes at $t=0$)



(a.) small time, L are like open circuits since I through L cannot instantaneously jump a finite value.

$\Rightarrow$

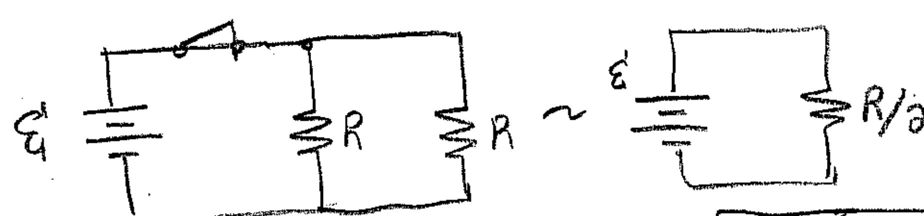


effective circuit at $t=0^+$

$$\therefore\ \boxed{I = \mathcal{E}/R}$$

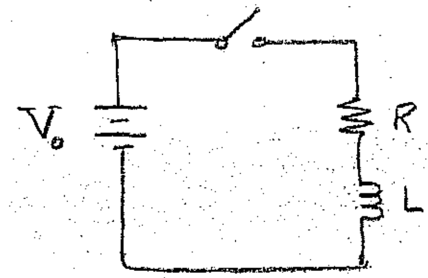
(b.) large times, L are like short circuits since they are no longer using energy to build up their B-fields.

$\Rightarrow$



$$\therefore\ \boxed{I = 2\mathcal{E}/R}$$

Problem 11 Assume that the inductor $L = 1.0\mu H$ is connected in series to a resistor $R = 1.0\,k\Omega$ and a voltage source $V_o = 7.0V$.



- (a.) find $I(t)$ supposing the current is zero at time zero.
- (b.) If at $t = 0$ we measure a voltage of $1.0V$ on the resistor then find the current in the circuit as a function of time for $t \geq 0$.

Kirchoff's Voltage Rule

$$V_o - IR - L\frac{dI}{dt} = 0$$

$$\Rightarrow \frac{dI}{dt} + \left(\frac{R}{L}\right)I = \frac{V_o}{L}$$

$$\Rightarrow I(t) = c_1 e^{-t/\tau} + \frac{V_o}{R} \quad \text{by calculus,} \quad \tau = L/R$$

(a.) $I(0) = 0 \Rightarrow c_1 + V_o/R = 0 \therefore C_1 = -V_o/R$

$$\therefore I(t) = \frac{V_o}{R}\left(1 - e^{-t/\tau}\right) \quad \text{where} \quad \tau = \frac{1.0\mu H}{1.0k\Omega} = 10^{-9}s$$

$$\Rightarrow \boxed{I(t) = (7mA)\left[1 - \exp(-10^9\, t/s)\right]} \qquad \downarrow \quad \frac{1}{\tau} = \frac{10^9}{s}$$

(b.) If $V_R = 1.0V \Rightarrow I = \frac{1.0V}{1.0k\Omega} = 1mA = I(0)$

$$I(0) = C_1 + \frac{V_o}{R} = 1mA \Rightarrow C_1 = 1mA - 7mA = -6mA.$$

$$\Rightarrow \boxed{I(t) = -6mA\exp(-10^9 t/s) + 7mA}$$

$\Big(I(t) = C_1 e^{-\frac{Rt}{L}} + \frac{V_o}{R}$ checks as sol^{n} since

$\frac{dI}{dt} = -C_1\left(\frac{R}{L}\right)e^{-\frac{Lt}{R}} \Rightarrow \frac{dI}{dt} + \frac{R}{L}I = \frac{R}{L}\frac{V_o}{R} = \frac{V_o}{L}\Big)$

Problem 12 Suppose $\vec{E} = E_o < 1, 2, 3 >$ and $\vec{B} = B_o < 2, 0, 3 >$ where $E_o = 10\,N/C$ and $B_o = 3.0mT$.

- (a.) what is the force on a charge $q = 2.0\,\mu C$ which has speed $v = 3.0\,m/s$ in the x-direction.
- (b.) calculate the Poynting vector and find the intensity of the electromagnetic field. (it's constant since the given fields are also constant)

(a.) $$\vec{F} = q\vec{V}\times\vec{B} + q\vec{E} = qv\,\hat{i}\times\vec{B} + q\vec{E}$$
$$= qv\,\hat{i}\times(2B_o\hat{i} + 3B_o\hat{k}) + q\vec{E}$$
$$= \underline{-3qvB_o\,\hat{j} + qE_o\,(\hat{i}+\hat{j}+\hat{k})}.$$

(b) $$\vec{S} = \frac{1}{\mu_o}(E_o<1,2,3>)\times(B_o<2,0,3>)$$
$$= \frac{E_oB_o}{\mu_o}\det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 0 & 3 \end{bmatrix}$$
$$= \underline{\left(\frac{1}{\mu_o}E_oB_o\right)(6\hat{i} + 3\hat{j} - 4\hat{k})}.$$

Problem 13 Suppose a linearly polarized electromagnetic wave is propogating in the x-direction and it has an electric field $\vec{E}(x,y,z,t) = (10N/C)\cos(kx - \omega t)\hat{k}$. Find the formula for $\vec{B}$ at (x,y,z,t). In which plane does the magnetic field oscilate?

$\vec{E}$ oscilates in xz-plane

need $\vec{E}\times\vec{B}$ in $\hat{i}$ - direction.

need $\vec{E}\perp\vec{B}$ since this is E&M wave,

$\Rightarrow$ $\vec{B}$ cannot be in $\hat{i}$ or $\hat{k}$ directions

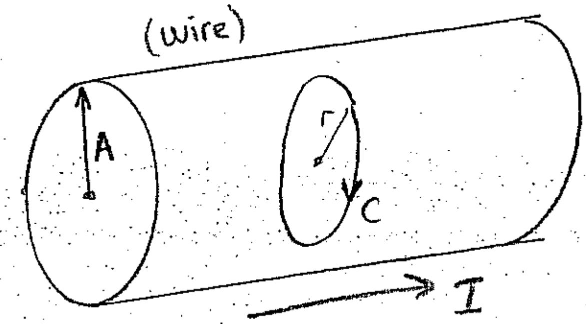
$\Rightarrow$ $\vec{B}$ must be in $\pm\hat{j}$ direction

note, $\vec{E}\times\vec{B} = EB\,\hat{k}\times(-\hat{j}) = EB\,\hat{i}$

$$\therefore\ \boxed{\vec{B}(x,y,z,t) = \left[(3.336\times10^{-8}T)\cos(kx-wt)\right](-\hat{j})}$$

$\vec{B}$ oscilates in xy-plane.

Problem 14 Suppose a uniformly distributed current I flows through a wire of radius A. If the wire is very long then find the magnetic field due to this current a distance r away from the center axis of the wire. Include multiple cases if need be.



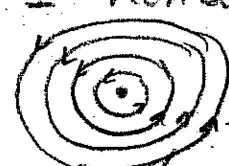
$\boxed{r \leq R}$ $\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$$B(2\pi r) = \mu_0 \left(\frac{\pi r^2}{\pi A^2} I \right)$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi A^2} \quad \text{for} \quad r \leq A}$$

$\boxed{r \geq R}$ amperian loop C contains all of I hence

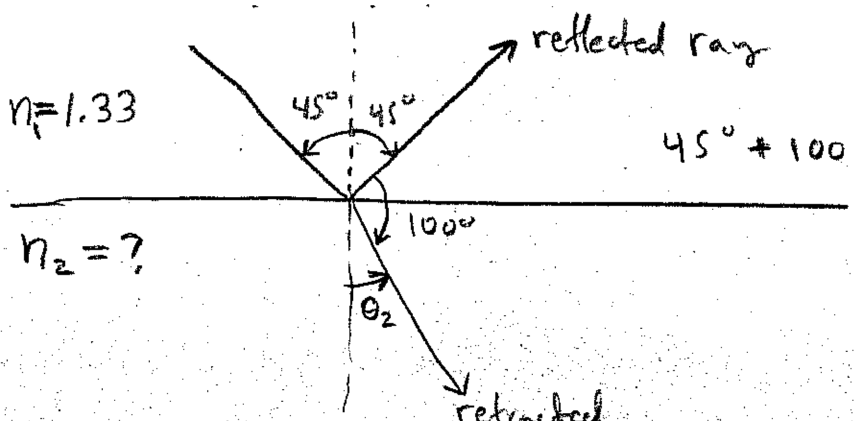
$B(2\pi r) = \mu_0 I \quad \therefore \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$



(in both cases the field circles the axis of wire

Some streamlines

Problem Light traveling in water (n = 1.33) strikes a glass plate at an angle of incidence of 45.0°. Part of the beam is reflected and part is refracted. (a) If the reflected and refracted portions make an angle of 100° with each other, what is the index of refraction of the glass?



$45° + 100° + \theta_2 = 180°$

$\Rightarrow \underline{\theta_2 = 35°}$.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{1.33 \sin 45°}{\sin 35°} = \boxed{1.64 = n_2}$$