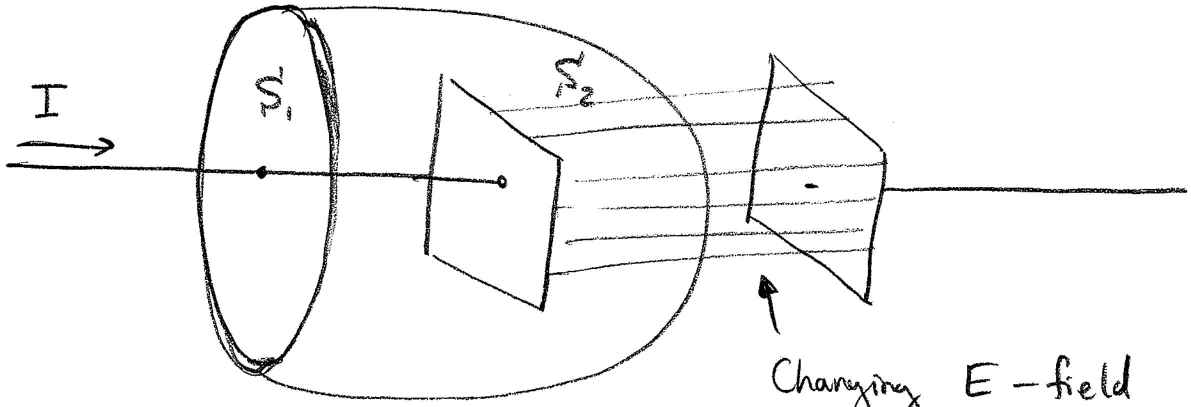
DISPLACEMENT CURRENT & MAXWELL'S EQ'NS 

Ampere's Law may fail when applied to non constant currents




$$\partial S_1 = \partial S_2 = C$$

$$\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{S_1} \neq \mu_0 \underbrace{I_{S_2}}$$

current through $S_1$ is $I$

zero current through $S_2$ HOWEVER there is a changing E-field through this $S_2$.

Thus, Maxwell suggested that the correct form of Ampere's Law is

$$\int_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_S + \mu_0 I_D$$

both through S

$$I_S = \int_S \vec{J} \cdot d\vec{A}$$

$$I_D = \int_S \epsilon_0 \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

Displacement "Current" $= \epsilon_0 \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{A} = \epsilon_0 \frac{\partial \Phi_E}{\partial t}$.

**Remark**: for the capacitor counter-example to naive Ampere's law one can show $I_{S_1} = I_D|_{S_2}$ See Tipler Example 30.1 pg. 1032

## The differential form of Ampere's Law with Maxwell's Correction

$$\int_{\partial S} \vec{B} \cdot d\vec{l} = \mu_o I_S + \mu_o I_D$$

$$= \mu_o \left( \int_S \vec{J} \cdot d\vec{A} + \epsilon_o \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{A} \right)$$

$$= \int_S \left[ \mu_o \vec{J} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t} \right] \cdot d\vec{A}$$

However, $\int_{\partial S} \vec{B} \cdot d\vec{l} = \int_S (\nabla \times \vec{B}) \cdot dA$ by Stoke's Th$^{\underline{m}}$. Hence, as this applies for essentially arbitrary surfaces $S'$. Finally we find Maxwell's Eq$^{\underline{n}}$'s

$$\boxed{\begin{aligned} \nabla \times \vec{B} &= \mu_o \vec{J} + \epsilon_o \mu_o \frac{\partial \vec{E}}{\partial t} \\ \nabla \cdot \vec{E} &= \rho/\epsilon_o \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \end{aligned}}$$

Electromagnetism expressed as a <u>local</u> theory

$$\vec{J} = \frac{\text{current}}{\text{area}}, \quad \rho = \frac{\text{charge}}{\text{volume}}$$

$$\vec{J} = \frac{d\vec{I}}{dA} \qquad \rho = \frac{dQ}{dV}$$

# Electromagnetic Waves in Vacuum

Assume $\vec{J} = 0$ and $\rho = 0$ hence

$$\nabla \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

Consider that,

$$\underbrace{\nabla \times (\nabla \times \vec{E})} = \nabla(\cancel{\nabla \cdot \vec{E}}^{\,0}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

$$\nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) = -\frac{\partial}{\partial t}\left(\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right) = -\epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\therefore \boxed{\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

WAVE EQ$^{\underline{n}}$!

identify as $\frac{1}{V^2} = \mu_0 \epsilon_0$

$$\therefore V = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

speed of light in vacuum.

LIGHT IS AN ELECTROMAGNETIC WAVE!

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# PROPERTIES OF LIGHT

- Should read your textbook and/or the power pts. we discussed in lecture. I'll just remind a few essential equations.

- The $\vec{E}$ & $\vec{B}$ in light are $\perp$ in vacuum.

- $\underbrace{E = cB}_{\text{magnitudes (not direction!)}}$ where $c$ = speed of light $= \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3\times10^8 \frac{m}{s}$.

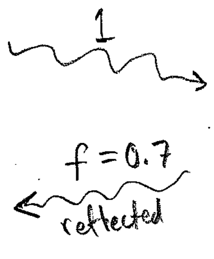
- $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$ ← Poynting vector, points in direction of motion of the Electromagnetic Wave.

- $\underbrace{P = \dfrac{S}{c} = \dfrac{1}{\mu_0 c}\|\vec{E}\times\vec{B}\|}_{\text{gives rise to radiation pressure from light}}$ = pressure of light

  Recall $P = \dfrac{\text{FORCE}}{\text{AREA}} = \dfrac{dF}{dA}$ (generally, but we'll probably only work constant $P$ problems)

**E87** (24.4 on p. 821 of Serway)
Consider laser pointer with 3.0 mW beam with area with 2.0 mm = diameter. Find radiation pressure on screen which reflects 70 % of light.

$$\text{Intensity} = \frac{\text{Power}}{\text{area}} = \frac{3.0\,mW}{\pi(1.0\,mm)^2} = 9.6\times10^2\,\frac{W}{m^2} \quad \left(\frac{\text{energy}}{(\text{area})(\text{time})}\right)$$




$$P_{avg} = \frac{S_{avg}}{c} + \frac{f\,S_{avg}}{c} = (1+f)\frac{S_{avg}}{c} = \frac{(1.7)(9.6\times10^2\,\frac{W}{m^2})}{3\times10^8\,m/s}$$

$$\Rightarrow \boxed{P = 5.4\times10^{-6}\,\frac{N}{m^2}}$$

Remark: The magnitude of $\vec{S}$ is $S = I =$ intensity this follows from dimensional analysis among other things.

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \sim \frac{c}{\mu_0}B^2$$

$$\Rightarrow \frac{1}{c}S \sim \frac{B^2}{2\mu_0} \sim \frac{\text{energy}}{\text{volume}}$$

$$\text{So } \frac{B^2}{2\mu_0}\cdot c \sim S \sim \frac{\text{energy}}{\text{volume}}\,\frac{\text{distance}}{\text{time}} \simeq \frac{\text{energy}}{\text{time}}\cdot\frac{\text{distance}}{\text{volume}}$$

$\therefore$ units for $\vec{S}$ are $\frac{\text{Power}}{\text{area}}$ which is intensity

This is one reason to include $\frac{1}{\mu_0}$ in def$^{\underline{n}}$ of $\vec{S}$. It forces $|\vec{S}| =$ intensity along $\hat{S}$.

Isotropic Radiation

spreads out in same way for all directions. If we assume conservation of energy in the wave then we can derive the inverse square law.

Malus' Law

$$\boxed{I = I_0 \cos^2\theta}$$

exit intensity

initial intensity

angle of linearly polarized light to the polarizer.

(see pg. 825 of Serway)