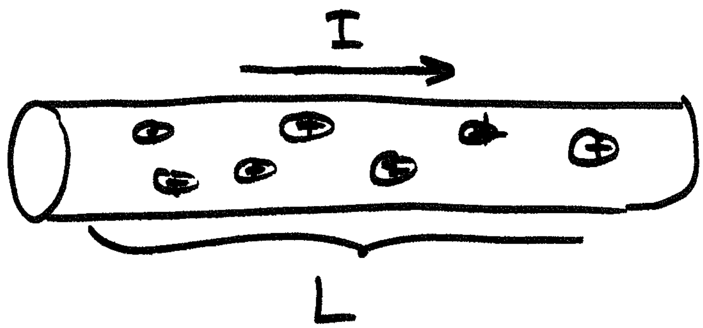
Lecture Notes

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Units for $|\vec{B}|$ are Teslas

$$1\ \mathrm{T} = \frac{\mathrm{N}}{\mathrm{C}\cdot \mathrm{m/s}}$$

1 Gauss ($G = 10^{-4}\ T$)



$$\vec{I} = q_{\text{total}}\,\vec{v}_{\text{drift}}$$

q_{total} proportional to l

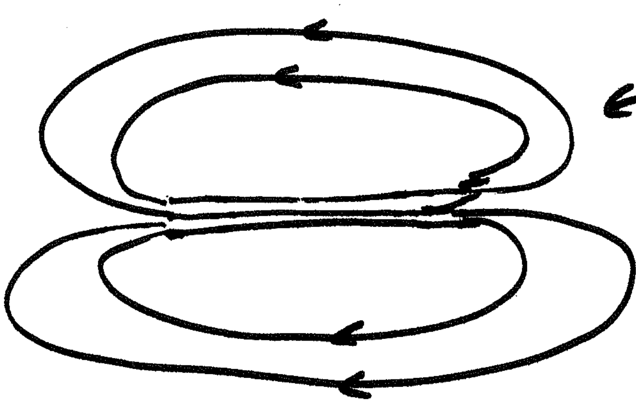
$$\vec{F} = I(\vec{l} \times \vec{B})$$
$$= l(\vec{I} \times \vec{B})$$

force on the length l of wire.

Infinitesimally,

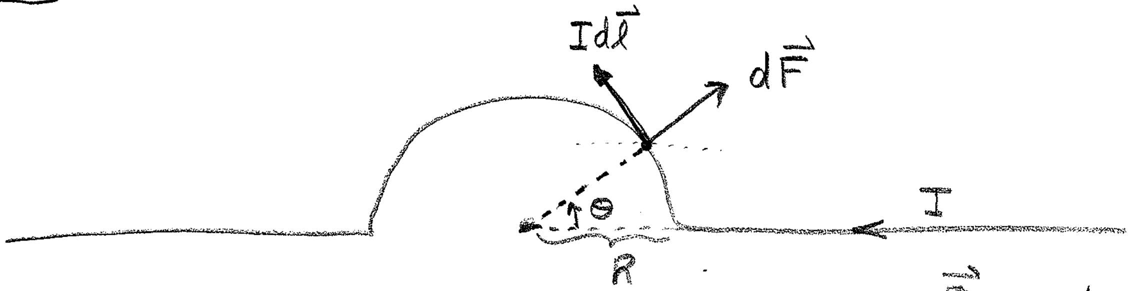
$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

force on $d\vec{l}$ due to $\vec{B}$



Loops.

E59 find Force on half-loop due to $\vec{B}$

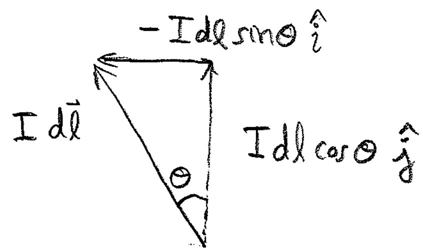


$\vec{B}$ out of page

$\vec{B} = B_0 \hat{k}$

1.) $d\vec{F} = I d\vec{l} \times \vec{B}$

2.)



$$\underline{d\vec{F} = \left(-I \sin\theta\, dl\, \hat{i} + I \cos\theta\, dl\, \hat{j}\right) \times \left(B_0 \hat{k}\right)}$$

3.) $\hat{i} \times \hat{k} = -\hat{j}$, $\hat{j} \times \hat{k} = \hat{i}$ and $dl = R\,d\theta$

$$d\vec{F} = -IR\sin\theta\, B_0\, d\theta \left(\hat{i} \times \hat{k}\right) + IR\cos\theta\, d\theta \left(\hat{j} \times \hat{k}\right) B_0$$

$$= \left(IR \sin\theta\, B_0\, d\theta\right)\hat{j} + \left(IR\cos\theta\, B_0\, d\theta\right)\hat{i}$$

4.) integrate for $0 \leq \theta \leq \pi$

$$\vec{F} = \left(IRB_0 \int_0^{\pi} \sin\theta\, d\theta\right)\hat{j} + \left(IRB_0 \int_0^{\pi} \cos\theta\right)\hat{i}$$

$$= \boxed{2IRB_0\, \hat{j}}$$

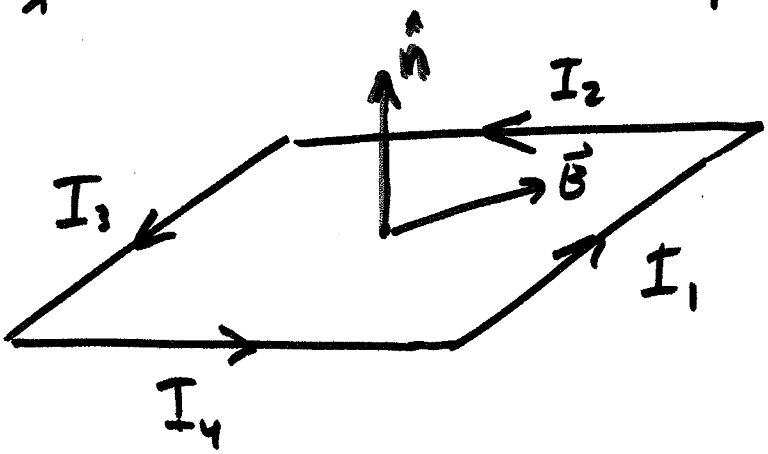
- velocity selector

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

need $v = \frac{E}{B}$ to balance

If $\vec{E} \perp \vec{B}$

- torque on current loop. (for axis through center of loop in plane of current)



$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

where $\vec{\mu} = NIA\hat{n}$

# of turns

current

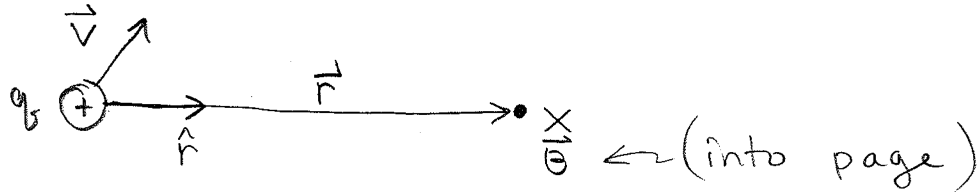
area of loop

normal (ccw)

choice.

Magnetic Field due to moving point charge

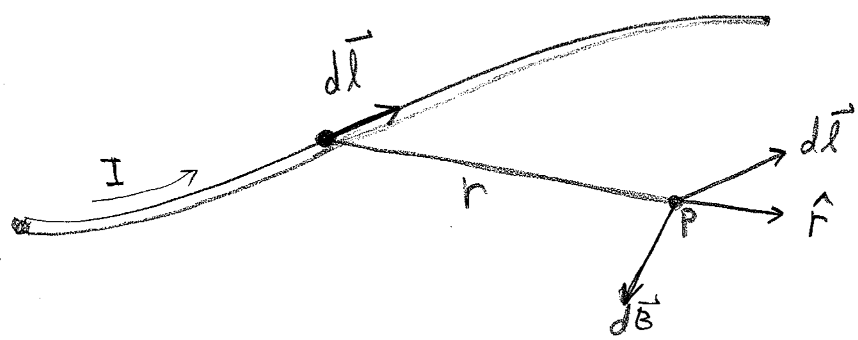
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \, \vec{v} \times \hat{r}}{r^2}$$



$$\mu_0 = 4\pi \times 10^{-7} \, \frac{T \cdot m}{A} = 4\pi \times 10^{-7} \, \frac{N}{A^2}$$

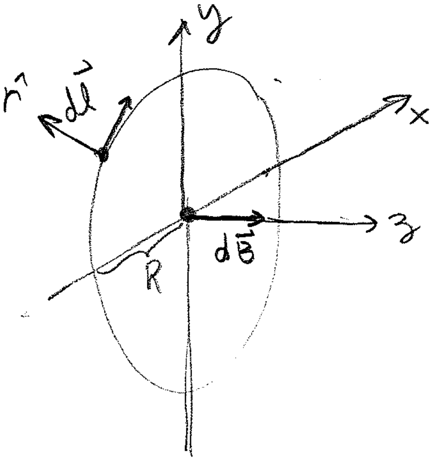
Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$



To find total $\vec{B}$ at P we have to add up all the $d\vec{B}$'s from each infinitesimal current $I d\vec{l}$.

E63 : find $\vec{B}$ due to a loop of current. Find $\vec{B}$ at the center of the loop.



Note: $\underbrace{\hat{r} \times d\vec{l} = dl\,\hat{k} = R\,d\theta\,\hat{k}}_{\text{for each pt. around loop always get } \hat{k}.}$

$$\vec{B} = \int_{\text{Loop}} d\vec{B} = \int_0^{2\pi} \frac{\mu_0}{4\pi} \underbrace{\frac{I R\, d\theta\, \hat{k}}{R^2}}$$

$$\frac{I\,d\vec{l} \times \hat{r}}{r^2} = \frac{R\,d\theta\,\hat{k}}{R^2}$$

(in this case)

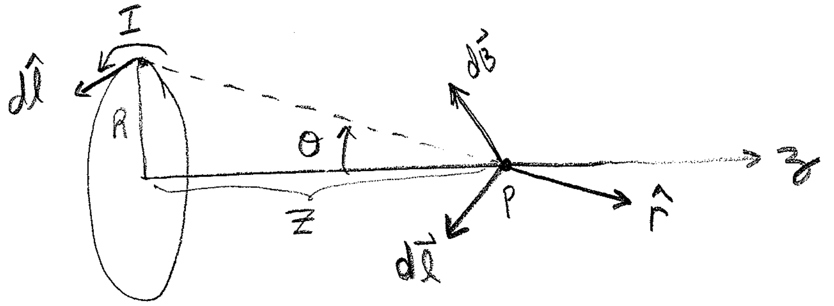
$$\therefore \quad \vec{B} = \frac{\mu_0 I (2\pi)\hat{k}}{4\pi R}$$

$$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I\,\hat{k}}{2R}}$$

current at center of loop.

If we had N-loops then $\vec{B} = \frac{\mu_0 N I}{2R}\hat{k}$ by almost the same arguments.

E64 Given loop of current find $\vec{B}$ at p.



$d\vec{l} \perp \hat{r}$

Apply Biot-Savart Law,

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I\,|d\hat{l}\times\hat{r}|}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl}{(z^2+R^2)}$$

Symmetry $\Rightarrow$ only $B_z \neq 0$.

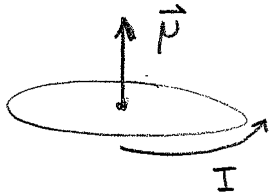
$$dB_z = \sin\theta\, dB = \frac{R}{\sqrt{z^2+R^2}}\,\frac{\mu_0 I\, dl/4\pi}{z^2+R^2}$$

$$B_z = \int_0^{2\pi} \frac{\mu_0 I R^2\, d\theta}{4\pi(z^2+R^2)^{3/2}}$$

$$\boxed{B_z = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}}$$

Remark: returning to $\boxed{E64}$ we found,

$$|\vec{B}| = B = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$



If $|z| \gg R$ then $(z^2+R^2)^{3/2} \to |z|^3$

Thus the far-field dipole strength is simply

$$B = \frac{\mu_0 I R^2}{2|z|^3}$$

$$= \frac{\mu_0}{4\pi}\,\frac{2I\pi R^2}{|z|^3}$$

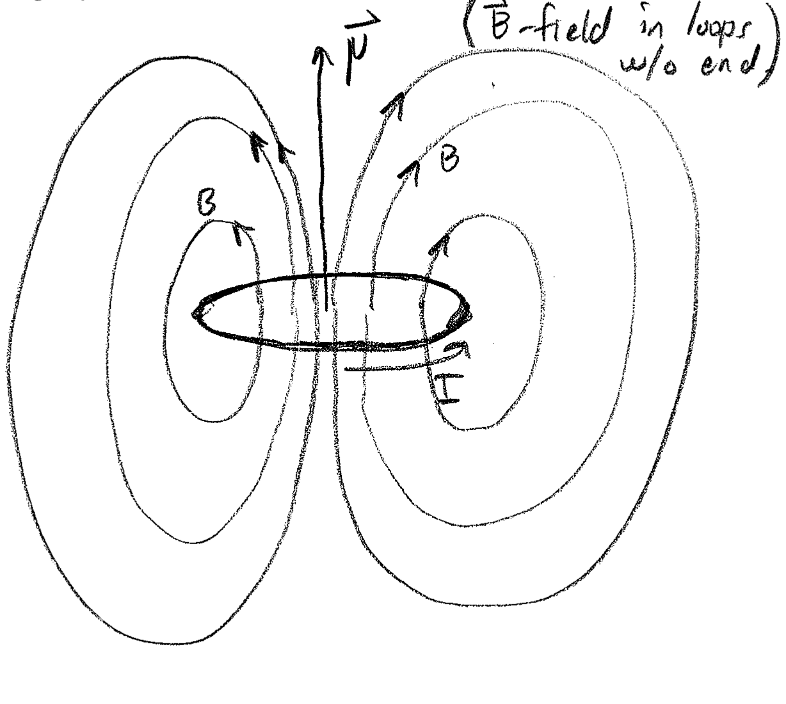
$$= \frac{\mu_0}{4\pi}\,\frac{2\mu}{|z|^3}$$

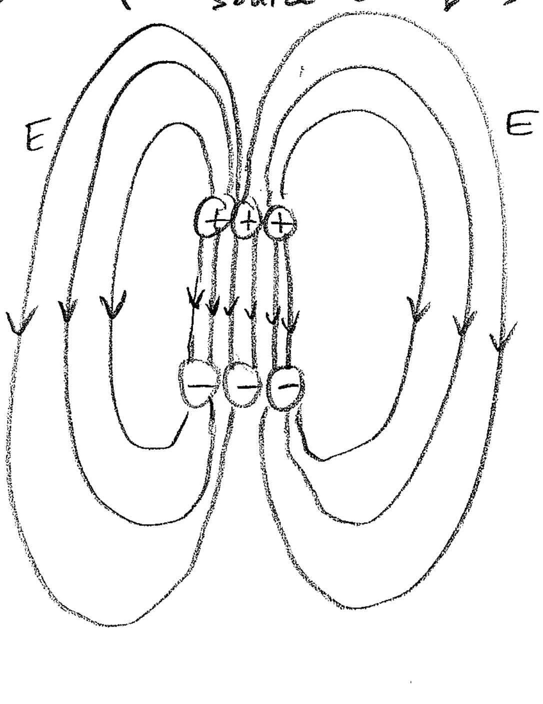
Def^{n}/ $\mu = \pi R^2 I$ ⇐ magnitude of <u>dipole</u> <u>moment</u> of current loop

The $\vec{E}$-field dipole was almost same. $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2d_q}{|z|^3}$

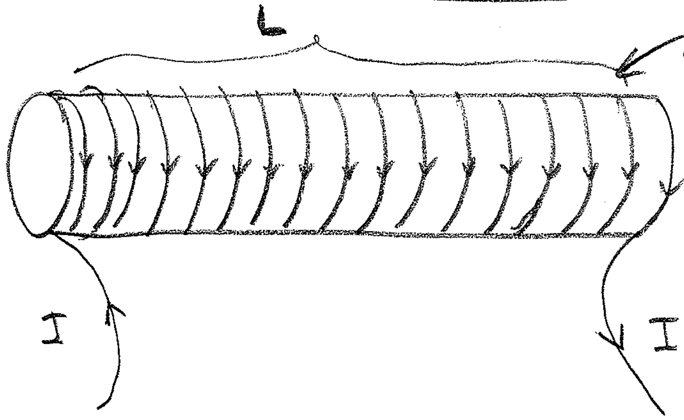
where d_q = <u>Electric dipole moment</u> (we didn't discuss much this semester)

($\vec{B}$-field in loops w/o end) ($\vec{E}$-field terminates on source charges)





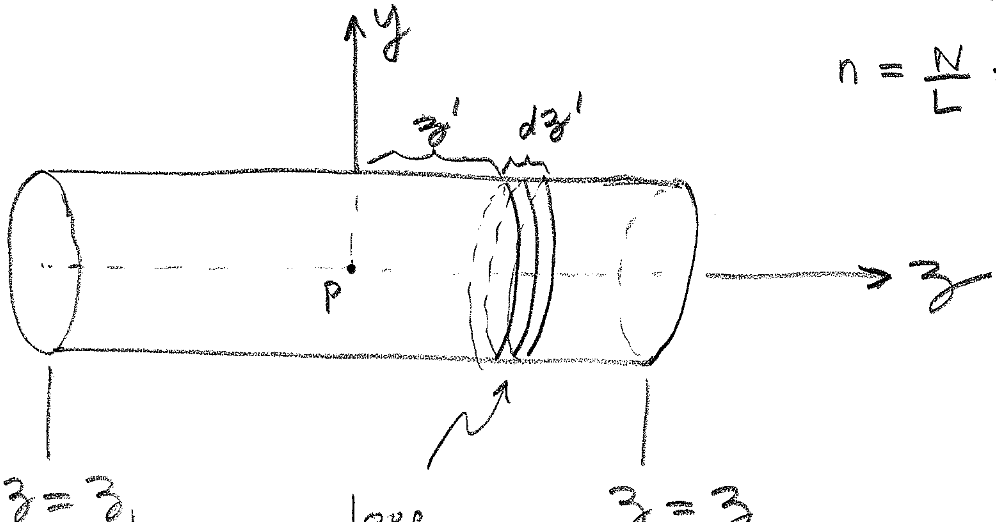
E65 $\vec{B}$ – due to solenoid



tightly wound wire. Current loops glued together intuitively.

$N = \#$ of turns for $z_1 \le z \le z_2$

$n = \frac{N}{L} = \frac{\text{turns}}{\text{length}}$



with

see E64

$$di = n I dz' = \left(\frac{\#\text{ turns}}{\text{length}}\right)(\text{current})(\text{length})$$ ✓

checking concept.

$$dB_z = \frac{\mu_0 R^2 n I dz'}{2\left[(z - z')^2 + R^2\right]^{3/2}}$$

$$\therefore \quad B_z = \int_{z_1}^{z_2} \frac{\mu_0 R^2 n I dz'}{2\left[(z - z')^2 + R^2\right]^{3/2}}$$

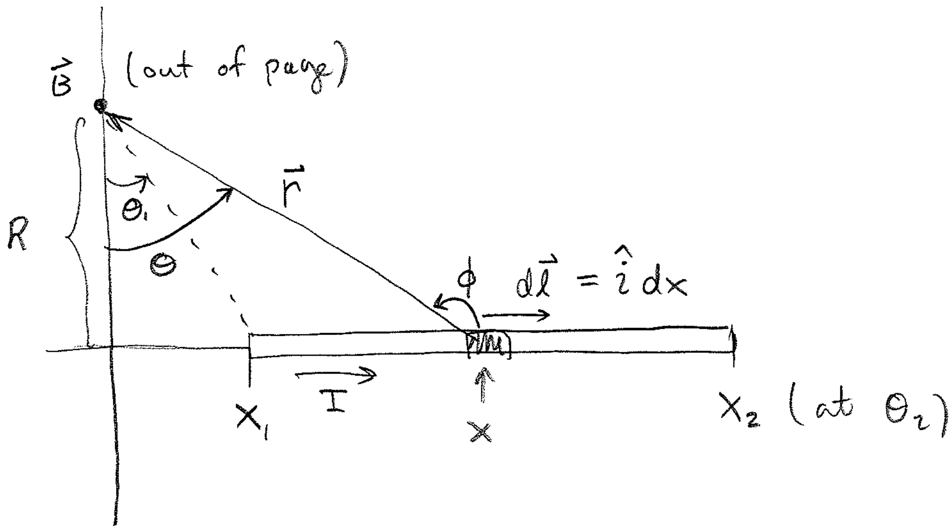
$$\boxed{B_z = \frac{\mu_0 n I}{2}\left(\frac{z - z_1}{\sqrt{(z - z_1)^2 + R^2}} - \frac{z - z_2}{\sqrt{(z - z_2)^2 + R^2}}\right)}$$

Magnetic field on <u>axis</u> of solenoid.

If $L \gg R$ then the solenoid is "long" and

$B_z \cong \mu_0 n I$ (center) $\quad B_z = \frac{1}{2}\mu_0 n I$ (ends)

E66 $\vec{B}$ due to segment of wire



$$dB = \frac{\mu_0}{4\pi}\frac{I\,dx}{r^2}\sin\phi$$

$$= \frac{\mu_0}{4\pi}\frac{I\,dx}{r^2}\cos\theta$$

<u>Note</u>: $x = R\tan\theta$

$dx = R\sec^2\theta\, d\theta$

$$= \frac{\mu_0}{4\pi}\frac{I}{r^2}R\sec^2\theta\cos\theta\, d\theta$$

$$= \frac{\mu_0 I R}{4\pi r^2}\sec\theta\, d\theta$$

$\cos\theta = \frac{R}{r}$

$r^2 = R^2/\cos^2\theta$

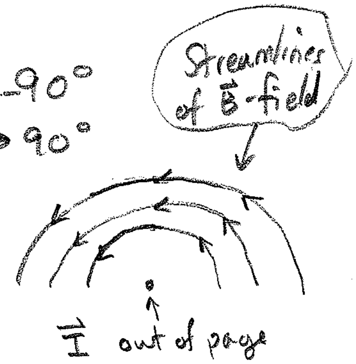
$$= \frac{\mu_0 I R}{4\pi}\frac{\cos^2\theta}{R^2}\sec\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi R}\cos\theta\, d\theta$$

$$B = \int_{\theta_1}^{\theta_2}\frac{\mu_0}{4\pi}\frac{I}{R}\cos\theta\, d\theta = \underline{\frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_2 - \sin\theta_1)}.$$

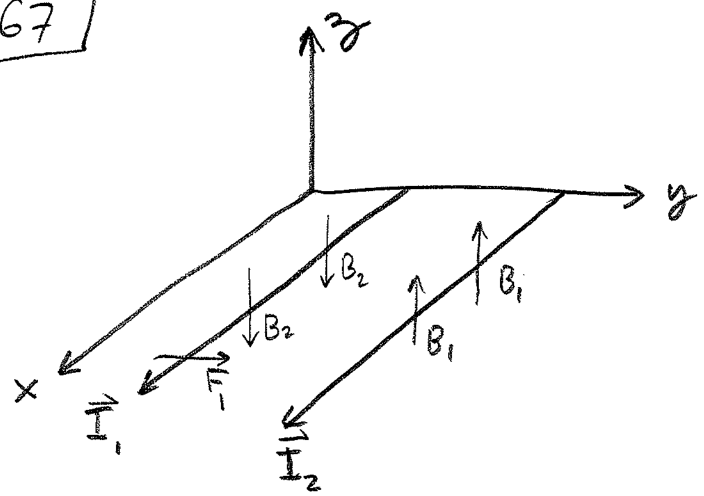
$$\Rightarrow \quad \underline{B = \frac{\mu_0 I}{2\pi R}} \quad \text{is} \quad \begin{array}{l}\theta_1 \to -90^\circ \\ \theta_2 \to 90^\circ\end{array}$$

$\vec{B}$ wraps cylindrically around wire as specified by RHR



Force due to parallel Currents

E67



$\vec{B}_2$ is field due to $\vec{I}_2$
$\vec{B}_1$ is field due to $\vec{I}_1$
(in my picture)

$$\vec{F}_1 = \vec{F}_{\substack{\text{on } I_1 \\ \text{over} \\ l_1}} = l_1 \vec{I}_1 \times \vec{B}_2 \quad (\text{in } +\hat{j}\text{-direction})$$

$$\vec{F}_2 = \vec{F}_{\substack{\text{on } I_2 \\ \text{over } l_1}} = l_1 \vec{I}_2 \times \vec{B}_1 \quad (\text{in } -\hat{j}\text{-direction})$$

$\therefore$ <u>parallel currents attract</u>.

E68 What if we reverse direction of $\vec{I}_2$?

E69 the force per unit length is found from

$$|d\vec{F}_2| = |I_2\, d\vec{l}_2 \times \vec{B}_1| = I_2\, dl_2\, B_1$$

$$\frac{dF_2}{dl_2} = I_2 B_1 \underset{\substack{\uparrow \\ \text{long wires}}}{\cong} \frac{\mu_0 I_1 I_2}{2\pi R}$$

Remark: Ampere defined by E67, E68, E69. This in turn defines the Coulomb