# FARADAY'S LAW, INDUCTION, CHANGING $\vec{B}$-fields

Upto this point we've primarily discussed cases where either the source charges don't move (electrostatics) or where the source currents are steady so the magnetic fields are unchanging (magnetostatics). We now turn to the question of what happens when we allow the magnetic field to change. It turns out this <u>induces</u> a change in an associated electric field (called the induced $\vec{E}$-field, or $E_{nc}$ by Tipler). (non conservative)

This induced electric field then causes charge to move and thus creates an <u>induced current</u>. When these things were discovered all of this was not understood. In fact, it was Maxwell who completed the story around 1850 when he wrote equations which completely describe how a changing $\vec{B}$-field creates a changing $\vec{E}$-field (and vice-versa). We'll explore Maxwell's Eq^ns later, but for now we begin by studying just one of them called "Faraday's Law"

Remark: the process I qualitatively outlined on 97 is known as <u>induction</u>. The voltage that gives $I_{\text{induced}}$ is called the emf (actually not a force!)

electro motive force

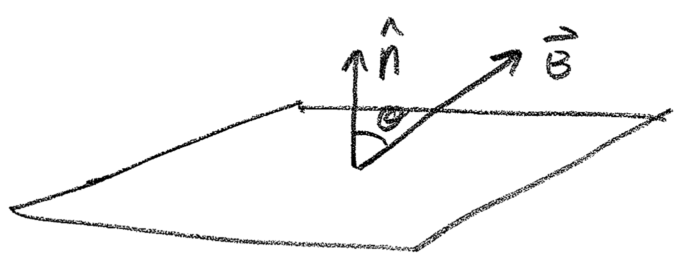
<u>FARADAY'S Law uses the magnetic flux</u>

$$\Phi_B = \int_S \vec{B} \cdot d\vec{S} = \text{\# of magnetic field lines that cut through } S.$$

Now, if $S$ is closed surface then this is easy to compute; $\Phi_B = 0$ because $\nexists$ magnetic monopoles (at least as far as we've observed)
However, if $S$ is not closed, so the boundary of $S$, which I denote $\partial S$, is some closed curve. We assume $S$ is <u>simply connected</u> so it has no holes.

<u>SPECIAL CASE</u>: if $\vec{B}$ is constant over $S$ then

$$\Phi_B = \int \vec{B} \cdot dA = \vec{B} \cdot \int d\vec{A} = \vec{B} \cdot \vec{A}$$
$$= \underline{BA\cos\Theta}.$$




However, if $S$ is curved or $\vec{B}$ is nonconstant then more calculation is req$^{d}$.

## FARADAY'S LAW

Suppose we have a simply connected surface $\vec{S}$ bounded by some curve $C = \partial S$ and $\Phi_B$ is the flux of the $\vec{B}$-field through $\vec{S}$ $\left(\Phi_B = \int_S \vec{B} \cdot d\vec{S}\right)$ then the emf (voltage) generated around the curve $C$ is as follows:

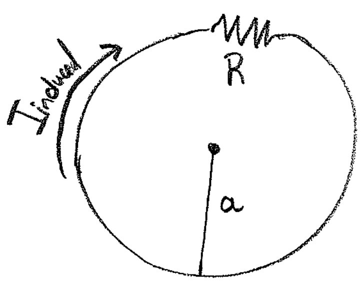
$$\boxed{\mathcal{E} = \frac{-d\Phi_B}{dt}}$$

The minus indicates that this emf ($\mathcal{E}$) induces a current in C which seeks to counter the change in $\Phi_B$. (THIS IS LENZ' LAW, THE REASON FOR (−))

E74) Suppose $\hat{k}$ points out of page. The $\vec{B}(t)$ is given.

CASE I) $\vec{B}(t) = \alpha t^2 \hat{k}$ (where $\alpha > 0$ is a dimensional constant to give $\vec{B}(t)$ correct units)

Find current in loop below given a CCW orientation for C.




$$\Phi_B(t) = (\text{AREA})(B(t)) = \pi a^2 \alpha t^2$$

$$\frac{d\Phi_B}{dt} = 2\pi a^2 \alpha t = -\mathcal{E}$$

The flux is increasing out of page $\Rightarrow$ induced current should give $B_{induced}$ into page $\Rightarrow$ CW current

$$\boxed{I_{induced}(t) = \frac{-\mathcal{E}}{R} = \frac{2\pi a^2 \alpha t}{R}}$$

Remark: $I_{induced}$ goes CW BUT the loop was CCW oriented. This is why I state $I_{induced} = -\mathcal{E}/R$. (which is positive since $\mathcal{E} = -2\pi a^2 \alpha t < 0$)

CASE II / $\vec{B}(t) = -\alpha t^2 \hat{k}$

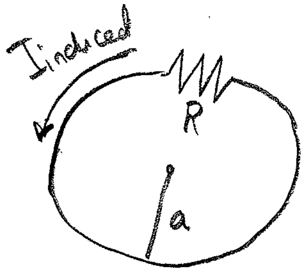
again circle radius $a$ given CCW orientation so $\hat{n} \parallel \hat{k}$ and we find $\Phi_B(t) = (B(t))(\text{AREA})$

$$\Rightarrow \Phi_B(t) = -\alpha\pi a^2 t^2$$

$$\Rightarrow \frac{d\Phi_B}{dt} = -2\alpha\pi a^2 t$$

$$\Rightarrow \underbrace{\mathcal{E} = 2\alpha\pi a^2 t}_{\text{this being positive}}$$

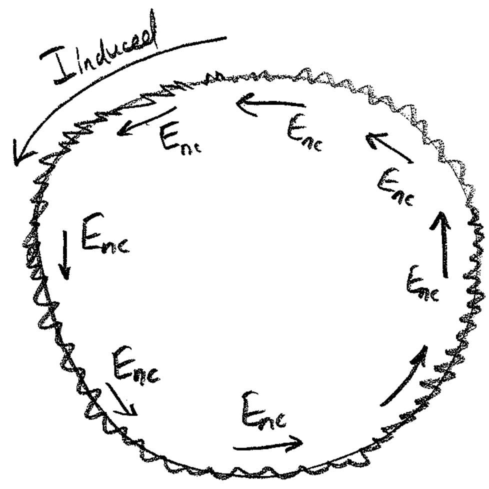
$\Longrightarrow$ induced current goes in the CCW (+) direction.




(flux $\Phi_B$ decreasing out of page $\Rightarrow$ $\Phi_{B_{induced}}$ out of page)

$$\boxed{I_{induced} = \frac{\mathcal{E}}{R} = \frac{2\pi\alpha a^2 t}{R}}$$

<u>CONCEPT</u>: usually we're not given a nice loop with a specific resistance. Usually the resistance of the loop is unknown or unclear in problems. One simple model to guide conceptual thought is that loops are made from a resistor bent into a loop (a real wire serves this purpose since metals do have <u>nonzero</u> resistivity)




As $\Phi_B$ changes it <u>induces</u> $E_{nc}$. Which induces $I_{induced}$

($E_{nc}$ <u>curls</u> around the loop due to $\frac{\partial B}{\partial t} \neq 0$)

# Mathematics of all this (for those who like calc. III)

Recall $\mathcal{E} = \oint_C \vec{E}_{nc} \cdot d\vec{l}$ (voltage produced from line integral of responsible $\vec{E}$ field)

FARADAY'S LAW SAYS (FROM EXPERIMENTS!)

$$\mathcal{E} = \underbrace{\oint_{C=\partial S} \vec{E}_{nc} \cdot d\vec{l}}_{\text{STOKE'S Th}^{m}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{S}$$

$$\iint_S (\nabla \times \vec{E}_{nc}) \cdot d\vec{S} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

regarding $t$ as independent variable like $x, y, z \ldots$

This holds for all surfaces which are topologically compatible with Stoke's Th$^{m}$ and that is enough to give us

$$\boxed{\nabla \times \vec{E}_{nc} = -\frac{\partial \vec{B}}{\partial t}}$$

The differential form of Faraday's Law.

Previously we also learned that

$$\nabla \cdot \vec{E} = \rho/\epsilon_0, \quad \nabla \cdot \vec{B} = 0$$

The remaining eq$^{n}$ for $\nabla \times \vec{B}$ involves Ampere's Law and something else... we'll save it for later.

— (End of Calc. III digression) —

## Faraday's Law for Many Loops

If we have a flux of $\Phi_B$ through $N$-loops then Faraday's Law is modified to

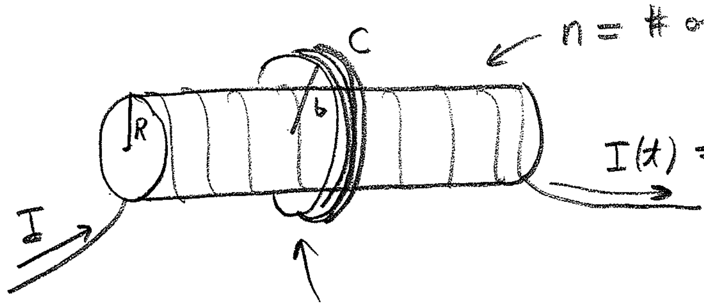
$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Lenz' Law

Again the (−) simply indicates the polarity of $\mathcal{E}$ is such that it counters the $d\Phi_B$ over $dt$.

**E75**

$n$ = # of turns/length




$I(t) = \underbrace{I_0 \cos(\omega t)}_{\text{given current through the Solenoid.}}$

N-loops of wire outside solenoid at $b > R$.

$I > 0 \Rightarrow \underbrace{\vec{B}_{solenoid}}_{\text{rightward}}$

**Flux Through C** : 
$$\Phi_B = \pi R^2 B(t)$$
$$= \pi R^2 \mu_0 n I(t)$$
$$= \underline{\pi R^2 \mu_0 n I_0 \cos(\omega t) = \Phi_B}$$

($\vec{B} = 0$ outside long solenoid)

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = \underline{-N\omega \pi R^2 \mu_0 n I_0 \sin(\omega t)}.$$

number of big loops around solenoid.

## Nice Examples



1.) Sliding Bar (see pg. 970 of Tipler) $\boxed{E76}$

2.) Moving loop through constant B (see p. 968 of Tipler) $\boxed{E77}$

3.) coin-flip charge storing (see p. 969 of Tipler) $\boxed{E78}$

(will add later)