EXAMPLES OF GEOMETRIC SERIES

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + \cdots = \frac{a}{1-r} \quad \text{iff } |r| < 1$$

"if and only if"

Alternatively,

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots = \frac{a}{1-r} \quad \text{provided } |r| < 1.$$

Notice, if $a_n = ar^n$ then $\dfrac{a_{n+1}}{a_n} = \dfrac{ar^{n+1}}{ar^n} = r$. Thus $\sum a_n$ is geometric implies $\dfrac{a_{n+1}}{a_n} = r$. This gives a quick check if $\sum a_n$ is geometric when in doubt.

1.) $\displaystyle\sum_{n=1}^{\infty} \frac{ne^n}{7^n}$ is <u>not</u> geometric since $a_n = \dfrac{ne^n}{7^n}$ has

$$\frac{a_{n+1}}{a_n} = \left(\frac{(n+1)\,e^{n+1}}{7^{n+1}}\right)\left(\frac{7^n}{ne^n}\right) = \left(\frac{n+1}{n}\right)\frac{e}{7} \quad \underline{\text{not}} \text{ a constant.}$$

2.)

$$\sum_{n=0}^{\infty} \pi^n\left(6^{-n} + 8^{-n+1}\right) = \sum_{n=0}^{\infty}\left(\frac{\pi}{6}\right)^n + 8\sum\left(\frac{\pi}{8}\right)^n$$

$$= \frac{1}{1-\pi/6} + \frac{8}{1-\pi/8}$$

$$= \boxed{\frac{6}{6-\pi} + \frac{64}{8-\pi}}$$

3.)

$$\sum_{n=2}^{\infty} x^n = \sum_{j=0}^{\infty} x^{j+2} \; : \; \boxed{\begin{array}{c} j = n-2 \\ n = j+2 \end{array}} \quad \text{(changed dummy index of } \Sigma\text{)}$$

$$= \sum_{j=0}^{\infty} x^2 x^j$$

$$= \boxed{\frac{x^2}{1-x}}$$

$a = x^2$
$r = x$
given $|x| < 1$