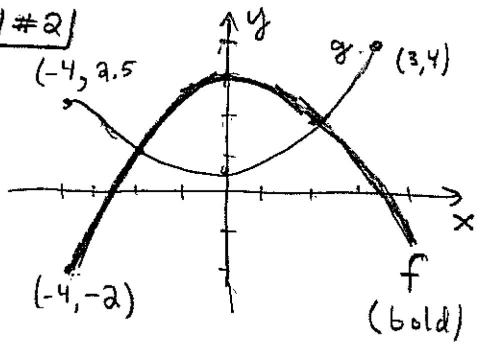
HOMEWORK 1 : §1.1 # 2, 8, 28, 30, 45, 50, 56, 64, 65 & §1.2 # 1, 6, 7, 8, 9 & §1.3 # 3, 31, 59, 63, 65, 66 : MATH 131, CALCULUS I, STEWART 6^{th} Ed.

§1.1#2



(a.) $f(-4) = -2$
$g(3) = 4$

(b.) $f(x) = g(x)$ at points of intersection $\Rightarrow \boxed{x = -2, 2}$

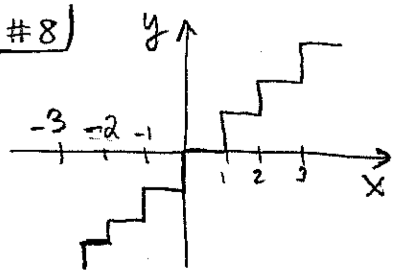
(c.) $f(x) = -1$ has sol^{n}'s
$x \cong 4$ or $x \cong -3$
note the graph isn't clear on this, my answers are only approximate, there is an uncertainty of about ± 0.2.

(d.) f is decreasing on $(0, 4)$

(e.) $\left.\begin{array}{l} \text{dom}(f) \cong [-4, 4] \\ \text{range}(f) \cong [-2, 3] \end{array}\right\}$ approximate

(f.) $\text{dom}(g) \cong [-4, 3]$
$\text{range}(g) \cong [0.5, 4]$

§1.1#8



this is not the graph of a function. It fails the vertical line test. Equivalently, there is more than one output for a particular input $(x = 0, \pm 1, \pm 2, \dots)$

§1.1#28 Let $f(x) = \dfrac{5x+4}{x^2+3x+2}$ define a function f. Observe

$$f(x) = \frac{5x+4}{(x+1)(x+2)} \Rightarrow \boxed{\begin{aligned} \text{dom}(f) &= \mathbb{R} - \{-1, -2\} \\ &= (-\infty, -2) \cup (-2, -1) \cup (-1, \infty) \end{aligned}}$$

(there are other notations also accepted. as I mentioned in lecture)

§1.1 #30 Let $g(u) = \sqrt{u} + \sqrt{4-u}$. The domain of g is given by $u \geq 0$ and $4-u \geq 0$ thus $0 \leq u \leq 4$ $\therefore$ $\boxed{\text{dom}(g) = [0, 4]}$

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§1.1 #45 The graph of the line segment from $(1,-3)$ to $(5,7)$ can be found from the two-point formula for a line

$$y = y_1 + \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Plug in $(1,-3) = (x_1, y_1)$ and $(5,7) = (x_2, y_2)$ to find,

$$\begin{aligned} y &= -3 + \left(\frac{7+3}{5-1}\right)(x-1) \\ &= -3 + \frac{10}{4}(x-1) \\ &= -3 + \frac{5}{2}(x-1) \end{aligned} \Rightarrow \boxed{\begin{array}{c} y = -3 + \frac{5}{2}(x-1) \\ 1 \leq x \leq 5 \end{array}}$$

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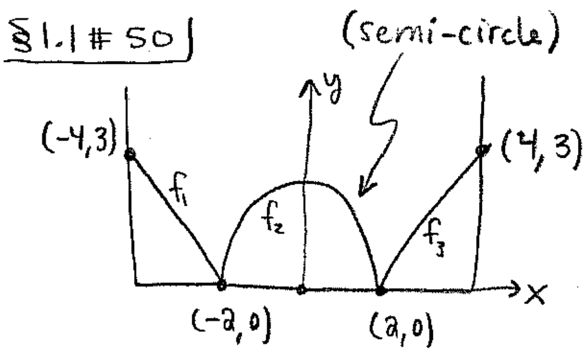
§1.1 #50



$$f_1(x) = 3 + \left(\frac{0-3}{-2+4}\right)(x+4) = 3 - \frac{3}{2}(x+4)$$

$$\underline{f_1(x) = -\frac{3}{2}x - 3} \quad (\text{for } -4 \leq x \leq -2)$$

Likewise, for $2 \leq x \leq 4$,

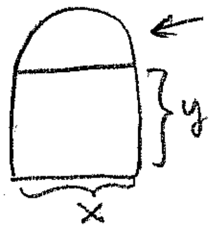
$$\underline{f_3(x) = 3 + \frac{3}{2}(x-4) = -3 + \frac{3}{2}x}$$

The circle has $x^2 + y^2 = 4 \Rightarrow f_2(x) = \sqrt{4-x^2}$ on $-2 < x < 2$. To summarize, this is the graph of a piece-wise defined function,

$$f(x) = \begin{cases} -\frac{3}{2}x - 3 & : \ x \leq -2 \\ \sqrt{4-x^2} & : \ -2 < x < 2 \\ -3 + \frac{3}{2}x & : \ x \geq 2 \end{cases}$$

<u>Remark:</u> since the function is continuous at $x = \pm 2$ we could use $-2 \leq x \leq 2$ and be correct.

§1.1#56) Given the perimeter is 30 find area as function of x.



$$30 = x + 2y + \tfrac{1}{2}\pi x \Rightarrow 2y = 30 - x - \tfrac{\pi}{2}x$$
$$\Rightarrow y = 15 - \tfrac{1}{2}\left(1+\tfrac{\pi}{2}\right)x$$

$$A = xy + \tfrac{1}{2}\pi\left(\tfrac{x}{2}\right)^2 = x\left[15 - \tfrac{1}{2}\left(1+\tfrac{\pi}{2}\right)x\right] + \tfrac{1}{8}\pi x^2$$
$$= 15x + \left(\tfrac{\pi}{8} - \tfrac{1}{2}\left(1+\tfrac{\pi}{2}\right)\right)x^2$$
$$= \boxed{15x - \left(\tfrac{1}{2} + \tfrac{\pi}{8}\right)x^2 = A}$$

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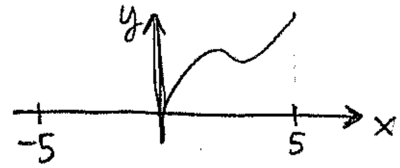
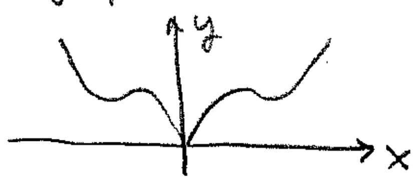
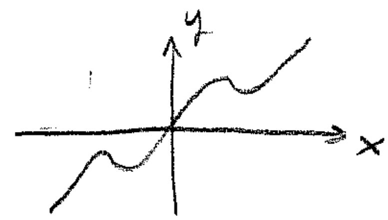
§1.1#64) A function has domain $[-5,5]$ and graph



(a.) If f is even then $f(-x) = f(x)$ and graph(f) mirrors over y-axis,



(b.) If f is odd then $f(-x) = -f(x)$ and the graph reflects through origin.



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§1.1#65) Let $f(x) = \frac{x}{x^2+1}$. Show f is odd. Notice that $\text{dom}(f) = (-\infty, \infty)$. Let $x \in \mathbb{R}$ and observe,

$$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -\left(\frac{x}{x^2+1}\right) = -f(x).$$

Therefore, f is an <u>odd</u> <u>function</u>.

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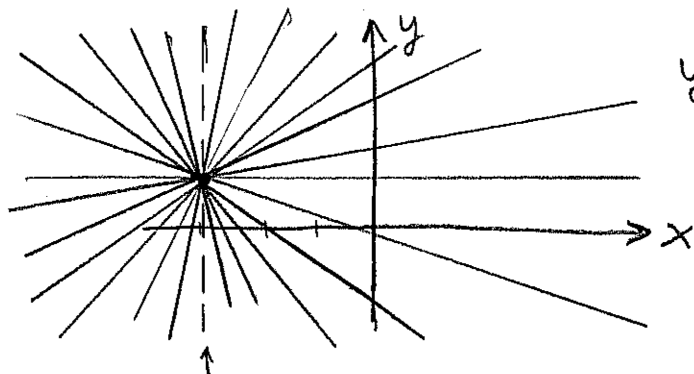
§1.2#1)

(a.) $f(x) = \sqrt[5]{x}$: root fnct.
(b.) $g(x) = \sqrt{1-x^2}$: algebraic fnct.
(c.) $h(x) = x^9 + x^4$: 9^{th} degree polymomial fnct.
} all algebraic fnct.
(d.) $r(x) = (x^2+1)/(x^3+x)$: rational fnct.
(e.) $s(x) = \tan(2x)$: trigonometric fnct.
(f.) $t(x) = \log_{10}(x)$: logarithmic fnct.
} transcendental fncts.

§1.2#6 What do all members of the family of functions $f_m(x) = 1 + m(x+3)$ have in common?

Since I know the point-slope formula for a line the answer clear: $y = f_m(x)$ is a line with slope m that goes through $(-3, 1)$.



$y = f_m(x)$ for various $m \in \mathbb{R}$.

$x = -3$ not in the family, but all other lines through $(-3,1)$ are in $\mathcal{F} = \{ f_m(x) \mid m \in \mathbb{R} \}$.

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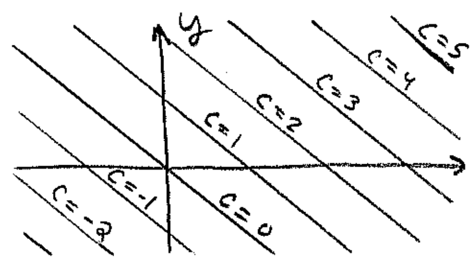
§1.2#7 What do all members of $f_c(x) = c - x$ have in common?

They all have slope $m = -1$. However, they have different y-intercepts $b = c$



$y = f_c(x)$ for various $c \in \mathbb{R}$.

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§1.2#8 Find quadratic polynomial with points $(-2, 2)$, $(0,1)$, $(1, -2.5)$ on its graph

Since it's a quadratic we know $f(x) = Ax^2 + Bx + C$. Furthermore, $f(0) = 1 \Rightarrow 1 = C \therefore \underline{f(x) = Ax^2 + Bx + 1}$.

Now, plug in the other data,

$$\left.\begin{aligned} f(-2) &= 4A - 2B + 1 = 2 \\ f(1) &= A + B + 1 = -2.5 \end{aligned}\right\}$$

Solve, Note $B = -3.5 - A$ then substitute into 1^{st} Eq^n

$4A - 2(-3.5 - A) + 1 = 2$

$\Rightarrow 6A = 2 - 1 - 7 = -6$

Therefore, $A = -1$, $B = -2.5$ and

$$\boxed{f(x) = -x^2 - 2.5x + 1}$$

$\boxed{\text{§1.2 #9] Find an expression for a cubic function } f \text{ if } f(1) = 6,\ f(-1) = f(0) = f(2) = 0}$

Happily we're given zeros! Apply wisdom of Factor Th$^{\underline{m}}$;

$$f(x) = A(x+1)x(x-2)$$

Then fix A by using $f(1) = A(2)(1)(-1) = 6 \Rightarrow \underline{A = -3}$

$$\therefore \quad \boxed{f(x) = -3x(x+1)(x-2)}$$

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§1.3 #3] See sol$^{\underline{n}}$ in text.

$\boxed{\text{§1.3 #31] Find (a.) } f \circ g, \text{ (b.) } g \circ f, \text{ (c.) } f \circ f, \text{ (d.) } g \circ g \text{ and their domains for } f(x) = x^2 - 1 \text{ and } g(x) = 2x+1}$

(a.) $(f \circ g)(x) = f(g(x)) = f(2x+1) = \boxed{(2x+1)^2 - 1 = (f \circ g)(x)}$

(b.) $(g \circ f)(x) = g(f(x)) = g(x^2-1) = \boxed{2(x^2-1) + 1 = (g \circ f)(x)}$

(c.) $(f \circ f)(x) = f(f(x)) = f(x^2-1) = \boxed{(x^2-1)^2 - 1 = (f \circ f)(x)}$

(d.) $(g \circ g)(x) = g(g(x)) = g(2x+1) = \boxed{2(2x+1)+1 = (g \circ g)(x)}$

Of course we can <u>simplify</u> the formulas as follows:

$$(f \circ g)(x) = 4x^2 + 4x = 4x(x+1)$$
$$(g \circ f)(x) = 2x^2 - 1$$
$$(f \circ f)(x) = x^4 - 2x^2 = x^2(x^2 - 2)$$
$$(g \circ g)(x) = 4x + 3$$

Clearly domains are <u>all</u> $(-\infty, \infty)$. I think $g \circ g$ is the interesting part, we found the composite of two lines is another line whose slope is a product of the constituent slopes; $4 = 2 \cdot 2$... which brings us to the next exercise §1.3 #59 ask us to prove this observation in general.

§1.3 #59 Let f, g be linear functions such that $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$. Show that $f \circ g$ is also linear with slope $m_1 m_2$.

$$\begin{aligned}(f \circ g)(x) &= f(m_2 x + b_2) \\ &= m_1(m_2 x + b_2) + b_1 \\ &= m_1 m_2 x + m_1 b_2 + b_1 \\ &= \underbrace{mx + b}\end{aligned}$$

line with slope $m = m_1 m_2$
and y-intercept $b = m_1 b_2 + b_1$.

<u>Remark</u>: this is where the chain-rule comes from.

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§1.3 #63 (a.) Suppose f and g are even functions. What can we say about $f+g$ and fg (b.) f, g odd same?

(a.) Assume f, g are even fncts, this means that $f(-x) = f(x)$ and $g(-x) = g(x)$. Consider,

$$(f+g)(-x) \underset{\uparrow}{=} f(-x) + g(-x) \underset{\uparrow}{=} f(x) + g(x) \underset{\uparrow}{=} (f+g)(x).$$

(def$^{\underline{n}}$ of $f+g$) since f, g even def$^{\underline{n}}$ of $f+g$ again.

Thus $f+g$ is an even fnct.
Likewise,

$$\begin{aligned}(fg)(-x) &= f(-x)\,g(-x) &&: \text{def}^{\underline{n}} \text{ of } fg. \\ &= f(x)\,g(x) &&: f, g \text{ even} \\ &= (fg)(x) &&: \text{def}^{\underline{n}} \text{ of } fg.\end{aligned}$$

Thus fg is an even function.

(b.) Assume f, g are odd functions,

$$\begin{aligned}(fg)(-x) &= f(-x)\,g(-x) &&: \text{def}^{\underline{n}} \text{ of } fg \\ &= (-f(x))(-g(x)) &&: f, g \text{ odd.} \\ &= f(x)\,g(x) &&: \text{minus signs cancel.} \\ &= (fg)(x) &&: \text{def}^{\underline{n}} \text{ of } fg \quad \therefore \underline{fg \text{ is odd}}.\end{aligned}$$

Proof that $f+g$ odd similar

§1.3#65 Suppose g is an even function and let $h = f \circ g$. Is $h \circ g$ always an even function?

Consider
$$\begin{aligned} h(-x) &= (f \circ g)(-x) \\ &= f(g(-x)) \\ &= f(g(x)) \\ &= h(x). \quad \therefore \ \underline{h \text{ is even}} \ (\text{always}) \end{aligned}$$

(Of course we assume dom(f) includes $g(x)$.)

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§1.3#66 Suppose g is an odd fnct. Let $h = f \circ g$ is h always an odd function? What if f is odd, what if f is even?

Assume g is odd fnct.

$$\begin{aligned} h(-x) &= (f \circ g)(-x) \\ &= f(g(-x)) && : \text{def}^{\underline{n}} \text{ of } f \circ g. \\ &= f(-g(x)) && : g \text{ is odd} \\ &= \begin{cases} -f(g(x)) & : \text{if } f \text{ odd} \\ f(g(x)) & : \text{if } f \text{ even} \\ f(-g(x)) & : \text{if } f \text{ neither odd nor even} \end{cases} \end{aligned}$$

Thus generally h is neither odd nor even.

However,

f is even $\Longrightarrow$ h is even.

f is odd $\Longrightarrow$ h is odd.

<u>For example</u>,

$g(x) = \sin(x)$

$f_1(x) = x^2$

$f_2(x) = x^3$

$\longrightarrow (f_1 \circ g)(x) = \sin^2(x)$: even.

$\longrightarrow (f_2 \circ g)(x) = \sin^3(x)$: odd.

<u>Remark</u>: It is not correct to assume a particular example to prove the general statement. Need to argue abstractly here.