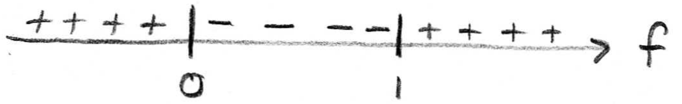
# Homework 24, Calculus I

§4.5 #9] Use calculus to graph $y = \frac{x}{x-1} = f(x)$

Note $\text{dom}(f) = (-\infty, 1) \cup (1, \infty)$ and $f(0) = 0$ thus,



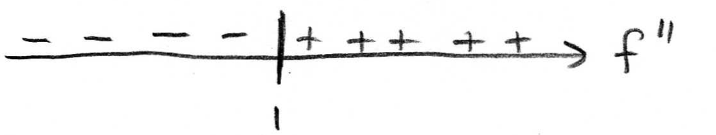
Calculate by quotient rule,

$$\frac{df}{dx} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$ no zeroes, V.A. at $x = 1$.

∴ f decreases everywhere except $x = 1$.

Next for concavity we need,

$$\frac{d^2f}{dx^2} = \frac{2}{(x-1)^3}$$ no zeroes, V.A. at $x = 1$

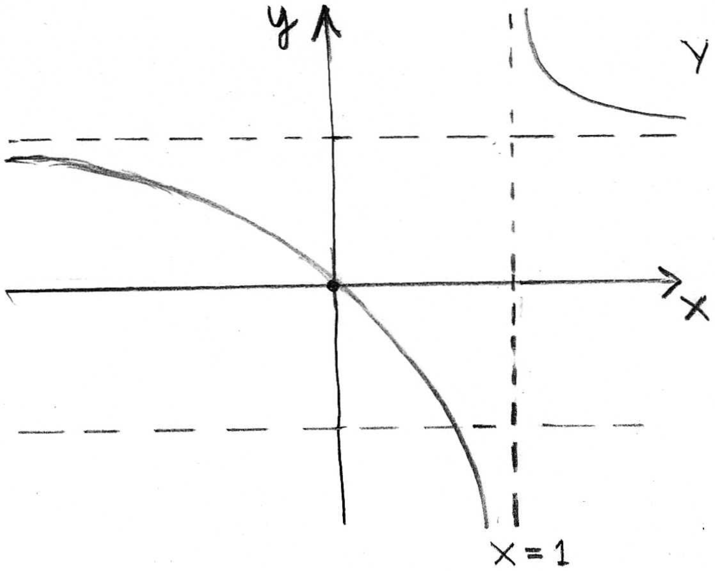


∴ f concave down $x < 1$
f concave up $x > 1$.

Horizontal asymptotes are $y = 1$ at $\pm\infty$ since

$$\lim_{x\to\pm\infty}\left(\frac{x}{x-1}\right) = \lim_{x\to\pm\infty}\left(\frac{1}{1-\underset{0}{\cancel{1/x}}}\right) = 1.$$

So we graph. We want an inflection "point" at 1



$$Y = \frac{x}{x-1} = 1 - \underbrace{\frac{1}{x-1}}$$

as you can see the graph is $Y = \frac{1}{x}$ moved up one and over one.

§4.5 #13

$y = \frac{x}{x^2+9} = f(x)$ , note dom$(f) = (-\infty, \infty)$ and we have $f(0) = 0$.

$$- - - \underset{0}{|} + + + \rightarrow f$$

We calculate,

$$\frac{df}{dx} = \frac{x^2+9-2x^2}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2} = \frac{(3-x)(3+x)}{(x^2+9)^2}$$

We have critical numbers (horizontal tangents) $x = \pm 3$,

$$- - - \underset{-3}{|} + + + + + \underset{3}{|} - - - \rightarrow \frac{df}{dx}$$

f increases on $(-3, 3)$
f decreases on $\mathbb{R} - (-3,3)$.

This means $f(-3) = \frac{-1}{6}$ is a local min and $f(3) = \frac{1}{6}$ is a local max.

$$\frac{d^2f}{dx^2} = \frac{-2x(x^2+9)^2 - 2(x^2+9)(2x)(9-x^2)}{(x^2+9)^4}$$

$$= \frac{-2x^3 - 18x - 36x + 4x^3}{(x^2+9)^3}$$

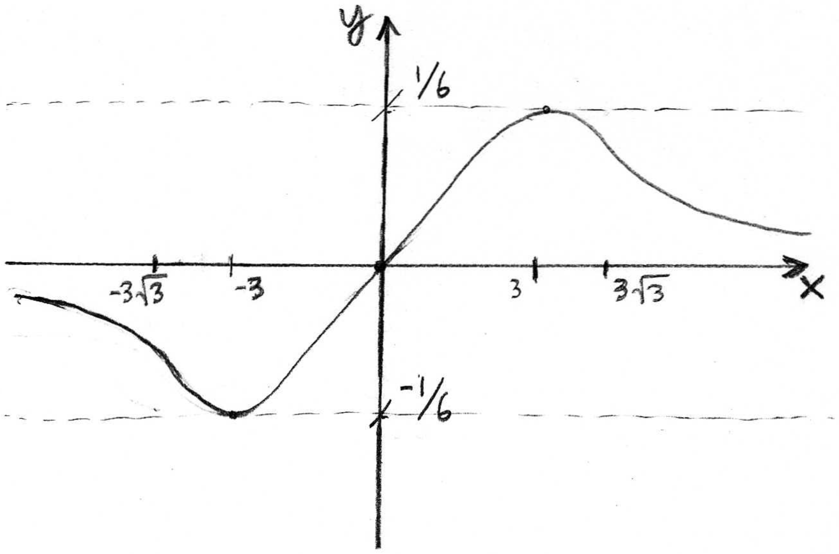
$$= \frac{-54x + 2x^3}{(x^2+9)^3} = \frac{2x(x^2-27)}{(x^2+9)^3} = 0 \rightarrow x = 0$$

$$\rightarrow x^2 - 27 = 0$$
$$x = \pm 3\sqrt{3}$$

$$- - - \underset{-3\sqrt{3}}{|} + + + + \underset{0}{|} - - - \underset{3\sqrt{3}}{|} + + + + \rightarrow$$

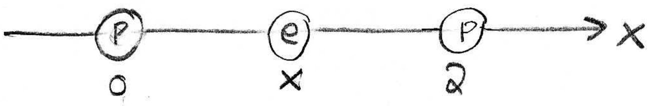
concave down $(-\infty, -3\sqrt{3})$, $(0, 3\sqrt{3})$
concave up $(-3\sqrt{3}, 0)$, $(3\sqrt{3}, \infty)$.

Notice $\lim_{x\to\pm\infty}\left(\frac{x}{x^2+9}\right) = \lim_{x\to\pm\infty}\left(\frac{1/x}{1+9/x^2}\right) = 0 \therefore$ $\underline{y = 0 \text{ horiz. tangent.}}$



To graph use all the data we've gathered about local max/min concavity and the H.A. $y = 0$.

§4.5#42



The force on e at position x with $0 < x < 2$,

$$F(x) = \frac{-ke^2}{x^2} + \frac{ke^2}{(x-2)^2} = ke^2\left(\frac{x^2-(x-2)^2}{x^2(x-2)^2}\right) = ke^2\left(\frac{4x-4}{x^2(x-2)^2}\right)$$

where $k, e > 0$ are constants. Graph the net force using calculus, what does this say about the force on the electron at x due to the protons at 0 and 2?

$$F = ke^2\left(\frac{1}{(x-2)^2} - \frac{1}{x^2}\right)$$

$$\frac{dF}{dx} = ke^2\left(\frac{-2}{(x-2)^3} + \frac{2}{x^3}\right)$$

$$= 2ke^2\left(\frac{-x^3+(x-2)^3}{(x-2)^3x^3}\right) \qquad (x-2)(x^2-4x+4) = x^3-4x^2+4x-2x^2+8x-8$$

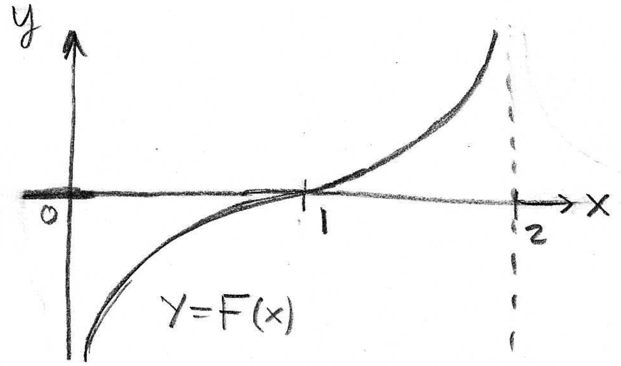
$$= 2ke^2\left(\frac{-x^3+x^3-6x^2+12x-8}{(x-2)^3x^3}\right)$$

$$= 2ke^2\left(\frac{-6x^2+12x-8}{(x-2)^3x^3}\right)$$

$$= -4ke^2\left(\frac{3x^2-6x+4}{(x-2)^3x^3}\right)$$

$\frac{dF}{dx} = 0$ for $3x^2-6x+4 = 0 \rightarrow x = \frac{6 \pm \sqrt{36-48}}{6}$, no real sol$^{\underline{n}}$

Thus the critical #'s are $x = 2$ and $x = 0$ where the force becomes infinitely large.



the force is zero at $x = 1$.

There is no local max/min on $(0,2)$ as you can see the force goes to ± ∞ at the endpoints.