Homework 25

§4.4#3 We can read from the graph that

(a.) $\lim_{x\to 2} f(x) = \infty$ (vertical asymptote)

(b.) $\lim_{x\to -1^-} f(x) = \infty$ (vertical asymptote, fnct. goes up)

(c.) $\lim_{x\to -1^+} f(x) = -\infty$ (vertical asymptote, fnct goes down

(d.) $\lim_{x\to \infty} f(x) = 1$ (horizontal asymptote $y=1$ as $x\to\infty$)

(e.) $\lim_{x\to -\infty} f(x) = 2$ (horiz. asymptote $y=2$ as $x\to -\infty$)

§4.4#7

$$\lim_{x\to\infty}\left(\frac{3x^2-x+4}{2x^2+5x-8}\right) = \lim_{x\to\infty}\left(\frac{3-\overset{\to 0}{\cancel{1/x}}+\overset{\to 0}{\cancel{4/x^2}}}{2+\underset{\to 0}{\cancel{5/x}}-\underset{\to 0}{\cancel{8/x^2}}}\right)$$: dividing numerator & denominator by x^2

$$= \lim_{x\to\infty}\left(\frac{3}{2}\right)$$: note $\frac{1}{x}$ and $\frac{1}{x^2} \to 0$ as $x\to\infty$.

$$= \boxed{\frac{3}{2}}$$

§4.4#9

$$\lim_{x\to\infty}\left(\frac{1}{2x+3}\right) = 0.$$ since $\frac{1}{\text{big \#}} \to 0$ as big # gets bigger & bigger.

§4.4#11

$$\lim_{x\to-\infty}\left(\frac{1-x-x^2}{2x^2-7}\right) = \lim_{x\to-\infty}\left(\frac{1/x^2-1/x-1}{2-7/x}\right) = \boxed{\frac{-1}{2}}$$

again the terms such as $1/x$ and $1/x^2 \to 0$ as $x\to-\infty$ since one over something large and negative gets very small and negative ... in the limit it goes to zero.

§4.4#19

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right) = \lim_{x\to\infty}\left(\frac{(\sqrt{9x^2+x}-3x)(\sqrt{9x^2+x}+3x)}{\sqrt{9x^2+x}+3x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}\right)$$

I divided by x both numerator & denominator

$$= \lim_{x\to\infty}\left(\frac{1}{\sqrt{9+1/x}+3/x}\right) = \frac{1}{\sqrt{9}} = \boxed{\frac{1}{3}}$$

§4.4 #22

$\lim_{x\to\infty} (\cos(x))$ d.n.e since $\cos(x)$ never settles to one value.

§4.4 #25

$$\lim_{x\to-\infty}(x^4+x^5) = \lim_{x\to-\infty}(x^5) = -\infty$$

for very large negative x the value of x^5 is much larger in magnitude than x^4.

§4.4 #29

$$\lim_{x\to\infty}\left(x\sin\left(\tfrac{1}{x}\right)\right) = \lim_{x\to\infty}\left(\frac{\sin(1/x)}{1/x}\right)$$

$$= \lim_{\theta\to 0^+}\left(\frac{\sin(\theta)}{\theta}\right)$$

$$= \boxed{1}$$

use substitution $\theta = \frac{1}{x}$ thus $\theta \to 0^+$ as $x \to \infty$.

: using limit from pg. 129 which we proved in class a while back.

Remark: later we will learn L'Hopitals Rule. That will allow us to solve many of these limits another way. Its not much better for these problems with the exception of #29.