Homework 28, Calculus I

§5.3 #15) Let $Y = \int_0^{\tan(x)} \sqrt{t+\sqrt{t}}\,dt$. Find $\frac{dY}{dx}$.

$$\frac{dY}{dx} = \frac{d}{dx}\int_0^{\tan(x)} \sqrt{t+\sqrt{t}}\,dt$$

$\longrightarrow$ Let $\int \sqrt{t+\sqrt{t}}\,dt = F(t)$. This exists because the integrand is continuous. I assume that x is suitably restricted so that $\tan(x) \geq 0$. Use FTC.

$$= \frac{d}{dx}\left[F(\tan(x)) - F(0)\right]$$

$$= \frac{dF}{dx}(\tan(x)) \cdot \frac{d}{dx}(\tan(x)) \quad : \text{ chain rule.}$$

$$= \boxed{\left(\sqrt{\tan(x)+\sqrt{\tan(x)}}\right)\sec^2(x)}$$

§5.3 #19) I use the FTC : $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$.

$$\int_{-1}^{2} (x^3 - 2x)\,dx = \left(\frac{1}{4}x^4 - x^2\right|_{-1}^{2}$$

← "evaluated from -1 to 2" means to do this

$$= \left(\frac{1}{4}(2)^4 - 2^2\right) - \left(\frac{1}{4}(-1)^4 - (-1)^2\right)$$

$$= \left(\frac{16}{4} - 4\right) - \left(\frac{1}{4} - 1\right)$$

$$= \boxed{\frac{3}{4}}$$

§5.3 #21)

$$\int_1^4 (5 - 2t + 3t^2)\,dt = \left(5t - t^2 + t^3\right|_1^4$$

$$= (20 - 16 + 64) - (5 - 1 + 1)$$

$$= \boxed{63}$$

§5.3 #23)

$$\int_0^1 x^{4/5}\,dx = \frac{5}{9}x^{\frac{9}{5}}\Big|_0^1 = \boxed{\frac{5}{9}}$$

§5.3 #25)

$$\int_1^2 \frac{3}{t^4}\,dt = \frac{3}{-3t^3}\Big|_1^2 = -\frac{1}{8} + 1 = \boxed{\frac{7}{8}}$$

§5.3 #27

$$\int_0^2 x(2+x^5)\,dx = \int_0^2 (2x + x^6)\,dx$$

$$= \left(x^2 + \frac{1}{7}x^7\right|_0^2$$

$$= 4 + \frac{128}{7} - 0$$

$$= \boxed{\frac{156}{7}}$$