Homework 29, Calculus I

§5.3 #29

$$\int_1^9 \frac{x-1}{\sqrt{x}}\,dx = \int_1^9 \left(x^{1/2} - x^{-1/2}\right)dx$$
$$= \left(\frac{2}{3}x^{3/2} - 2x^{1/2}\right|_1^9$$
$$= \left(\frac{2}{3}(\sqrt{9})^3 - 2\sqrt{9}\right) - \left(\frac{2}{3} - 2\right)$$
$$= (18-6) - \left(-\frac{4}{3}\right)$$
$$= 12 + \frac{4}{3}$$
$$= \boxed{40/3}$$

§5.3 #31

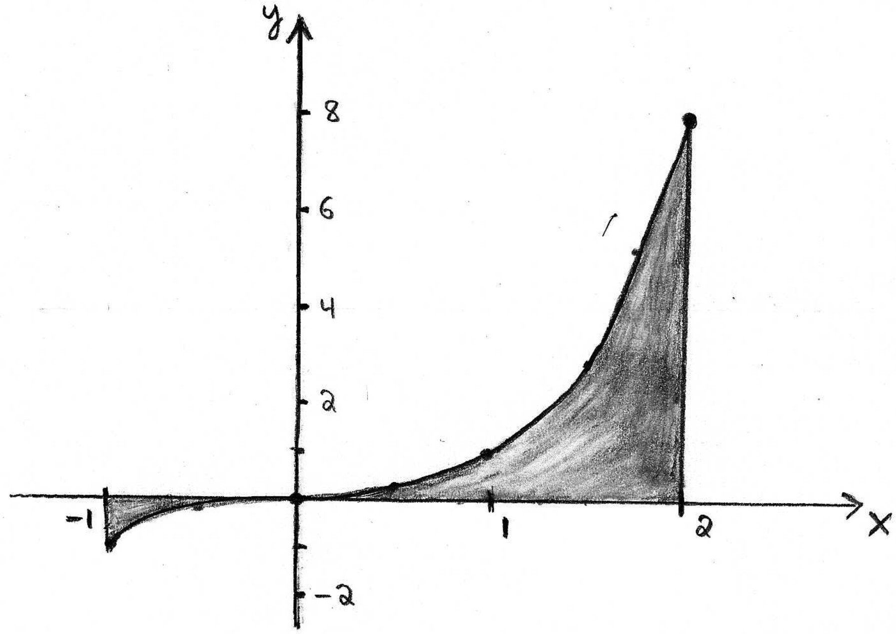
$$\int_0^{\pi/4} \sec^2 t\,dt = \tan t \Big|_0^{\pi/4} \qquad \left(\text{Recall, } \frac{d}{dt}(\tan t) = \sec^2 t.\right)$$
$$= \tan(\pi/4) - \tan(0)$$
$$= \boxed{1}$$

§5.3 #33

$$\int_1^2 (1+2y)^2\,dy = \int_1^2 (1 + 4y + 4y^2)\,dy$$
$$= \left(y + 2y^2 + \frac{4}{3}y^3\right|_1^2$$
$$= \left(2 + 8 + \frac{32}{3}\right) - \left(1 + 2 + \frac{4}{3}\right)$$
$$= \frac{62}{3} - \frac{13}{3}$$
$$= \boxed{49/3}$$

§5.3 #45 Integrate and illustrate the geometry implicit within the calculation.

$$\int_{-1}^{2} x^3\,dx = \frac{1}{4}x^4\Big|_{-1}^{2}$$
$$= \frac{1}{4}\left(2^4 - (-1)^4\right)$$
$$= \boxed{15/4}$$



the area above the x-axis is $\int_0^2 x^3\,dx = 8$ the area below the x-axis is $\left|\int_{-1}^{0} x^3\,dx\right| = \frac{1}{4}$ but that's counted as negative by the integral giving $8 - \frac{1}{4}$, the <u>signed</u> <u>area</u> under $y = x^3$ on $[-1, 2]$.

§5.3#69

$$\int_1^9 \frac{1}{2x}\,dx = \frac{1}{2}\ln|x|\Big|_1^9$$
$$= \frac{1}{2}\left(\ln(9) - \ln(1)\right)$$
$$= \ln(9^{1/2}) - \frac{1}{2}(0)$$
$$= \boxed{\ln(3)}$$

<u>Remark:</u> we must put absolute value bars for $\int \frac{1}{x}dx = \ln|x| + c$ because $x < 0$ is a possibility and it is true that

$$\frac{d}{dx}\left(\ln|x|\right) = \frac{d}{dx}\left(\ln\left(\sqrt{x^2}\right)\right) = \frac{1}{\sqrt{x^2}}\,\frac{1}{2\sqrt{x^2}}\cdot 2x = \frac{x}{|x|^2} = \frac{1}{x}.$$

Thus $\int \frac{1}{x}dx = \ln|x| + c$ is the correct integral.

§5.3#71)

$$\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}}\,dt = 6\sin^{-1}(t)\Big|_{1/2}^{\sqrt{3}/2}$$

$$= 6\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - 6\sin^{-1}\left(\frac{1}{2}\right)$$

$$= 6\left(\frac{\pi}{3}\right) - 6\left(\frac{\pi}{6}\right)$$

$$= \boxed{\pi}$$

Remark: $\sin^{-1}(\sqrt{3}/2) = \theta \to \sin(\theta) = \sqrt{3}/2 \to \theta = \frac{\pi}{3}$

$\sin^{-1}(1/2) = \theta \to \sin(\theta) = \frac{1}{2} \longrightarrow \theta = \frac{\pi}{6}$

that's how I think about these. However, I do allow scientific calculators on tests.

§5.3#73

$$\int_{-1}^{1} e^{u+1}\,du = \int_{-1}^{1} e\,e^u\,du$$

$$= e\int_{-1}^{1} e^u\,du$$

$$= e\left(e^1 - e^{-1}\right)$$

$$= e\left(e - \frac{1}{e}\right)$$

$$= \boxed{e^2 - 1}$$