Homework 2: §7.1 # 1, 2, 11, 17, 21, 22, 23, 25 // §7.6 # 1, 2, 10, 11 : Stewart 6^{th} Ed.

§7.1#1 (a.) f is 1-1 iff $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in dom(f)$.

(b.) f is 1-1 if it passes the horizontal line test.

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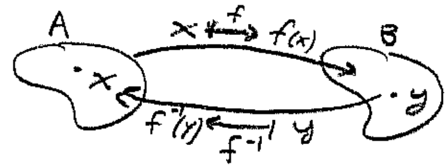
§7.2#2

(a.) If $f: A \longrightarrow B$ is a 1-1 fnct with range$(f) = f(A) = B$ then we define $f^{-1}(y) = x$ iff $f(x) = y$. Moreover, range$(f^{-1}) = A$ and dom$(f^{-1}) = B$.

(b.) There are many ways. One method,

1.) write $y = f(x)$
2.) solve for $x = g(y)$
3.) identify $f^{-1}(y) = g(y)$.



(c.) The graph$(f^{-1}) = \{(a, b) \mid (b, a) \in \text{graph}(f)\}$ you reflect across the line $y = x$.

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§7.1#11 Prove $g(x) = 1/x$ is 1-1

Let $a, b \in dom(g)$, meaning $a, b \neq 0$ and suppose

$$g(a) = g(b) \Rightarrow \frac{1}{a} = \frac{1}{b} \Rightarrow a = b.$$ <u>Thus, g is injective.</u>

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§7.1#17 If f is one-one and $f(2) = 9$ then f^{-1} exists and we calculate $f^{-1}(f(2)) = f^{-1}(9) \Rightarrow \boxed{f^{-1}(9) = 2}$

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§7.1#21 The formula $C = \frac{5}{9}(F - 32)$ where $F \geq -459.67$ expresses C as a fnct. of F where C is degrees Celsius and F is degrees Farenheight. Find F as a function of C and find its domain.

Notice $C = \frac{5}{9}(F - 32) \Rightarrow \frac{9}{5}C = F - 32 \Rightarrow \boxed{F = 32 + \frac{9}{5}C}$

If $F \geq -459.67$ then $C \geq \frac{5}{9}(-459.67 - 32) = -273.15$

thus $\boxed{F = g(C) = 32 + \frac{9}{5}C \text{ and } dom(g) = [-273.15, \infty)}$

<u>Remark:</u> Physicists might object to equality at absolute zero.

§7.1 #22 In special relativity the effective mass of a particle with rest mass m_o and <u>speed</u> v $(v \geq 0)$ is given by

$$m = f(v) = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

Find $f^{-1}(m)$ and interpret.

Let $m = \dfrac{m_o}{\sqrt{1-v^2/c^2}} \Rightarrow m^2 = \dfrac{m_o^2}{1 - v^2/c^2}$

$$\Rightarrow m^2 - \frac{m^2 v^2}{c^2} = m_o^2$$

$$\Rightarrow \frac{m^2}{c^2} v^2 = m^2 - m_o^2$$

$$\Rightarrow v = \frac{c}{m}\sqrt{m^2 - m_o^2}$$

$$\Rightarrow \boxed{v = c\sqrt{1 - (m_o/m)^2}}$$

Here c is the speed of light in vacuum. Notice $m \longrightarrow m_o$ we find $v \longrightarrow 0$ whereas $m \longrightarrow \infty$ we find $v \longrightarrow c$. Notice that for $v > c$ we would need $m = i\lambda$, such imaginary mass has yet to find physicists general approval. (unless you count STAR TREK)

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§7.1 #23 Let $f(x) = 3 - 2x$. Find $f^{-1}(y)$.

$$y = 3 - 2x \Rightarrow x = \frac{3-y}{2} \quad \therefore \quad \boxed{f^{-1}(y) = \frac{3-y}{2}}$$

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§7.1 #25 Let $f(x) = \sqrt{10-3x}$ find $f^{-1}(y)$

$$y = \sqrt{10-3x} \Rightarrow y^2 = 10 - 3x \Rightarrow x = \frac{-1}{3}(y^2 - 10)$$

$$\Rightarrow \boxed{f^{-1}(y) = \frac{10 - y^2}{3}}$$

§7.6 #1

(a.) $\sin^{-1}(\sqrt{3}/2) = y \Rightarrow \sin y = \frac{\sqrt{3}}{2} \Rightarrow \underline{y = \pi/3}$

(b.) $\cos^{-1}(-1) = z \Rightarrow \cos z = -1 \Rightarrow \underline{z = \pi}$.

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§7.6 #2

(a.) $\tan^{-1}(1/\sqrt{3}) = \theta \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1/2}{\sqrt{3}/2} \Rightarrow \underline{\theta = \pi/6}$.

(b.) $\sec^{-1}(2) = \beta \Rightarrow \sec\beta = 2$

$\Rightarrow \cos\beta = \frac{1}{2} \Rightarrow \underline{\beta = \pi/3}$.

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§7.6 #10 Skipped.

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§7.6 #11 Prove that $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$

Let $\theta = \sin^{-1}(x)$ then $\sin\theta = x$.
The Pythagorean Th$^{\underline{m}}$ states $\sin^2\theta + \cos^2\theta = 1$
thus,

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$\Rightarrow \underline{\cos(\sin^{-1}(x)) = \sqrt{1-x^2}} \;//$$

Question: why did we get to choose the + square-root? Generally, $\cos^2\theta = 1 - \sin^2\theta \Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$. Why no minus?
Recall range$(\sin^{-1}(x)) = [-\pi/2, \pi/2]$. What can you say about $\cos\theta$ on $[-\pi/2, \pi/2]$?