Homework Quiz : §5.6 #7 1/23/06

$$\int \underbrace{x^2}_{u} \underbrace{\sin(\pi x)\,dx}_{dv} = x^2\left(\frac{-1}{\pi}\cos(\pi x)\right) - \int \frac{-1}{\pi}\cos(\pi x)\, 2x\,dx$$

$u = x^2$
$du = 2x\,dx$
$dv = \sin \pi x$
$v = \frac{-1}{\pi}\cos \pi x$

$$= \frac{-1}{\pi}x^2\cos(\pi x) + \frac{1}{\pi}\left(\int \underbrace{2x}_{u} \underbrace{\cos(\pi x)\,dx}_{dv}\right)$$

$$= -\frac{x^2}{\pi}\cos(\pi x) + \frac{1}{\pi}\left(2x\frac{1}{\pi}\sin(\pi x) - \int \frac{1}{\pi}\sin \pi x \cdot 2dx\right)$$

$u = 2x$
$du = 2dx$
$dv = \cos \pi x\,dx$
$v = \frac{1}{\pi}\sin \pi x$

$$= -\frac{x^2}{\pi}\cos(\pi x) + \frac{2x}{\pi^2}\sin(\pi x) - \frac{2}{\pi^2}\int \sin(\pi x)dx$$

$$= \frac{-x^2}{\pi}\cos(\pi x) + \frac{2x}{\pi^2}\sin(\pi x) - \frac{2}{\pi^2}\left(\frac{-1}{\pi}\cos(\pi x)\right) + C$$

$$= \boxed{\cos(\pi x)\left(\frac{2}{\pi^3} - \frac{x^2}{\pi}\right) + \sin(\pi x)\left(\frac{2x}{\pi^2}\right) + C}$$

If you're wondering,

$$\int \sin(\pi x)dx = \int \sin(u)\frac{du}{\pi} = \frac{-1}{\pi}\cos(u) + C = \frac{-1}{\pi}\cos(\pi x) + C$$

$$\int \cos(\pi x)\,dx = \int \cos(u)\frac{du}{\pi} = \frac{1}{\pi}\sin(u) + C = \frac{1}{\pi}\sin(\pi x) + C$$

Both follow from $u = \pi x$ substitution. I used both of these in the calculation above.