Section 7.7

\# 7. $4y'' + y = 0$

$\Rightarrow$ the characteristic eqn. is

$4\lambda^2 + 1 = 0 \Rightarrow \lambda = \pm \frac{1}{2} i$

$\therefore$ the general solution to the D.E. is

$y(x) = c_1 \cos(\frac{1}{2}x) + c_2 \sin(\frac{1}{2}x)$

\# 21 $y'' - 2y' - 3y = 0$

The characteristic eqn. is

$\lambda^2 - 2\lambda - 3 = 0$

$\Rightarrow (\lambda - 3)(\lambda + 1) = 0 \Rightarrow \lambda = 3$ or $\lambda = -1$

$\therefore$ the general soln. to the D.E. is

$y(x) = c_1 e^{3x} + c_2 e^{-x}$

Apply initial conditions:

$y(1) = 3 \Rightarrow c_1 e^3 + c_2 e^{-1} = 3$

$y'(1) = 1 \Rightarrow 3c_1 e^3 - c_2 e^{-1} = 1$

$\Big\} \overset{*}{\Rightarrow}$ $c_1 = \frac{1}{e^3}$, $c_2 = 2e$

$\therefore$ the desired soln. is $y(x) = \frac{1}{e^3} e^{3x} + (2e) e^{-x}$

<u>Remark</u>: the algebra * can be done as follows:

$$\begin{array}{rl} & c_1 e^3 + c_2 e^{-1} = 3 \\ + & 3c_1 e^3 - c_2 e^{-1} = 1 \\ \hline & 4c_1 e^3 = 4 \end{array}$$

(adding equations, yes we can do it.)

$\Rightarrow c_1 e^3 = 1 \Rightarrow \boxed{c_1 = 1/e^3}$

then substitute into 1st eq^n: $\frac{1}{e^3} e^3 + c_2 e^{-1} = 3$

$1 + c_2/e = 3$

$\Rightarrow \boxed{c_2 = 2e}$