

Homework 13, Calculus I, Fall 2008§3.4#1

$$\begin{aligned}\frac{d}{dx} (3x^2 - 2\cos(x)) &= 3 \frac{d}{dx}(x^2) - 2 \frac{d}{dx}(\cos(x)) \\ &= \boxed{6x + 2\sin(x)}\end{aligned}$$

§3.4#3

$$\begin{aligned}\frac{d}{dx} (\sin(x) + \frac{1}{2}\cot(x)) &= \frac{d}{dx}(\sin(x)) + \frac{1}{2} \frac{d}{dx}(\cot(x)) \\ &= \cos(x) + \frac{1}{2} \frac{d}{dx} \left[ \frac{\cos(x)}{\sin(x)} \right] \\ &= \cos(x) + \frac{1}{2} \left[ \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} \right] \\ &= \cos(x) - \frac{1}{2} \left[ \frac{1}{\sin^2(x)} \right] \\ &= \boxed{\cos(x) - \frac{1}{2} \csc^2(x)}\end{aligned}$$

§3.4#5

$$\begin{aligned}g'(t) &= \frac{d}{dt} [t^3 \cos t] \\ &= \left[ \frac{d}{dt}(t^3) \right] \cos t + t^3 \frac{d}{dt}[\cos t] \\ &= \boxed{3t^2 \cos t - t^3 \sin t}\end{aligned}$$

§3.4#7

$$\begin{aligned}h'(\theta) &= \frac{d}{d\theta} (\theta \csc \theta - \cot \theta) \\ &= \frac{d\theta}{d\theta} \csc \theta + \theta \frac{d}{d\theta}(\csc \theta) - \frac{d}{d\theta}(\cot \theta) \\ &= \boxed{\csc \theta - \theta \csc \theta \cot \theta + \csc^2 \theta}\end{aligned}$$

Remark: we just saw  $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

and I showed in lecture  $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$   
Again its easy to show,

$$\frac{d}{dx}(\csc(x)) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{-\cos(x)}{\sin^2(x)} = \frac{-1}{\sin(x)} \left( \frac{\cos(x)}{\sin(x)} \right) = -\csc(x)\cot(x).$$

§ 3.4 #11] Lets calculate  $\frac{d}{d\theta}(\sec \theta)$  before we attempt  $\frac{d}{d\theta}\left(\frac{\sec \theta}{1+\sec \theta}\right)$ . (2)

$$\begin{aligned}\frac{d}{d\theta}(\sec \theta) &= \frac{d}{d\theta}\left(\frac{1}{\cos \theta}\right) \\ &= \frac{-(-\sin \theta)}{\cos^2 \theta} \\ &= \underline{\sec \theta \tan \theta}.\end{aligned}$$

Using this result we find,

$$\begin{aligned}\frac{d}{d\theta}\left[\frac{\sec \theta}{1+\sec \theta}\right] &= \frac{\left(\frac{d}{d\theta}(\sec \theta)\right)(1+\sec \theta) - \sec \theta \frac{d}{d\theta}(1+\sec \theta)}{(1+\sec \theta)^2} \\ &= \frac{\sec \theta \tan \theta (1+\sec \theta) - \sec \theta (\sec \theta \tan \theta)}{(1+\sec \theta)^2} \\ &= \frac{\sec \theta \tan \theta + \sec^2 \theta \tan \theta - \sec^2 \theta \tan \theta}{(1+\sec \theta)^2} \\ &= \boxed{\frac{\sec \theta \tan \theta}{(1+\sec \theta)^2}}.\end{aligned}$$

§ 3.4 #15]

$$\begin{aligned}\frac{d}{d\theta}[\sec \theta \tan \theta] &= \frac{d}{d\theta}\left[\frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta}\right] \quad \text{(chain-rule)} \\ &= \frac{d}{d\theta}\left[\frac{\sin \theta}{\cos^2 \theta}\right] \\ &= \frac{\cos \theta \cos^2 \theta - \sin \theta (-2\cos \theta \sin \theta)}{\cos^4 \theta} \\ &= \frac{1}{\cos^3 \theta} [\cos^2 \theta + 2\sin^2 \theta] \\ &= \frac{1}{\cos^3 \theta} [1 + \sin^2 \theta] = \boxed{\frac{1}{\cos^3 \theta} + \frac{\sin^2 \theta}{\cos^3 \theta}}\end{aligned}$$

$$\begin{aligned}\frac{d}{d\theta}(\cos^2 \theta) &= 2\cos \theta \frac{d}{d\theta}[\cos \theta] \\ &= -2\cos \theta \sin \theta\end{aligned}$$

Alternatively:

$$\begin{aligned}\frac{d}{d\theta}[\sec \theta \tan \theta] &= \frac{d}{d\theta}[\sec \theta] \tan \theta + \sec \theta \frac{d}{d\theta}[\tan \theta] \\ &= \sec \theta \tan \theta \tan \theta + \sec \theta \sec^2 \theta \\ &= \underline{\sec \theta + \tan^2 \theta + \sec^3 \theta}.\end{aligned}$$

Can you see these answers are equivalent?