Homework 13, Calculus I, FALL 2008

§3.4#1)

$$\frac{d}{dx}\left(3x^2 - 2\cos(x)\right) = 3\frac{d}{dx}\left(x^2\right) - 2\frac{d}{dx}\left(\cos(x)\right)$$
$$= \boxed{6x + 2\sin(x)}$$

§3.4#3)

$$\frac{d}{dx}\left(\sin(x) + \frac{1}{2}\cot(x)\right) = \frac{d}{dx}\left(\sin(x)\right) + \frac{1}{2}\frac{d}{dx}\left(\cot(x)\right)$$
$$= \cos(x) + \frac{1}{2}\frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right]$$
$$= \cos(x) + \frac{1}{2}\left[\frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)}\right]$$
$$= \cos(x) - \frac{1}{2}\left[\frac{1}{\sin^2(x)}\right]$$
$$= \boxed{\cos(x) - \frac{1}{2}\csc^2(x)}$$

§3.4#5)

$$g'(t) = \frac{d}{dt}\left[t^3\cos t\right]$$
$$= \left[\frac{d}{dt}(t^3)\right]\cos t + t^3\frac{d}{dt}\left[\cos t\right]$$
$$= \boxed{3t^2\cos t - t^3\sin t}$$

§3.4#7)

$$h'(\theta) = \frac{d}{d\theta}\left(\theta\csc\theta - \cot\theta\right)$$
$$= \frac{d\theta}{d\theta}\csc\theta + \theta\frac{d}{d\theta}\left(\csc\theta\right) - \frac{d}{d\theta}\left(\cot\theta\right)$$
$$= \boxed{\csc\theta - \theta\csc\theta\cot\theta + \csc^2\theta}$$

Remark: we just saw $\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$ and I showed in lecture $\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$. Again its easy to show,

$$\frac{d}{dx}\left(\csc(x)\right) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{-\cos(x)}{\sin^2(x)} = \frac{-1}{\sin(x)}\left(\frac{\cos(x)}{\sin(x)}\right) = -\csc(x)\cot(x).$$

§3.4 #11) Lets calculate $\frac{d}{d\theta}(\sec\theta)$ before we attempt $\frac{d}{d\theta}\left(\frac{\sec\theta}{1+\sec\theta}\right)$.

$$\frac{d}{d\theta}(\sec\theta) = \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)$$
$$= \frac{-(-\sin\theta)}{\cos^2\theta}$$
$$= \underline{\sec\theta\tan\theta}.$$

Using this result we find,

$$\frac{d}{d\theta}\left[\frac{\sec\theta}{1+\sec\theta}\right] = \frac{\left(\frac{d}{d\theta}(\sec\theta)\right)(1+\sec\theta) - \sec\theta\frac{d}{d\theta}(1+\sec\theta)}{(1+\sec\theta)^2}$$
$$= \frac{\sec\theta\tan\theta(1+\sec\theta) - \sec\theta(\sec\theta\tan\theta)}{(1+\sec\theta)^2}$$
$$= \frac{\sec\theta\tan\theta + \cancel{\sec^2\theta\tan\theta} - \cancel{\sec^2\theta\tan\theta}}{(1+\sec\theta)^2}$$
$$= \boxed{\frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}}.$$

§3.4 #15)

(chain-rule)

$$\frac{d}{d\theta}(\cos^2\theta) = 2\cos\theta\frac{d}{d\theta}[\cos\theta] = -2\cos\theta\sin\theta$$

$$\frac{d}{d\theta}[\sec\theta\tan\theta] = \frac{d}{d\theta}\left[\frac{1}{\cos\theta}\frac{\sin\theta}{\cos\theta}\right]$$
$$= \frac{d}{d\theta}\left[\frac{\sin\theta}{\cos^2\theta}\right]$$
$$= \frac{\cos\theta\cos^2\theta - \sin\theta(-2\cos\theta\sin\theta)}{\cos^4\theta}$$
$$= \frac{1}{\cos^3\theta}\left[\cos^2\theta + 2\sin^2\theta\right]$$
$$= \frac{1}{\cos^3\theta}\left[1+\sin^2\theta\right] = \boxed{\frac{1}{\cos^3\theta} + \frac{\sin^2\theta}{\cos^3\theta}}$$

Alternatively:

$$\frac{d}{d\theta}[\sec\theta\tan\theta] = \frac{d}{d\theta}[\sec\theta]\tan\theta + \sec\theta\frac{d}{d\theta}[\tan\theta]$$
$$= \sec\theta\tan\theta\tan\theta + \sec\theta\sec^2\theta$$
$$= \underline{\sec\theta\tan^2\theta + \sec^3\theta}.$$

Can you see these answers are equivalent?