Homework "Quiz" 9, 9/8/08

$\boxed{\S 3.1 \# 10}$

(a.) find slope of tangent to $y = 1/\sqrt{x}$ at point where $x = a$,

$$f'(a) = m = \lim_{x \to a}\left(\frac{f(x) - f(a)}{x - a}\right)$$

$$= \lim_{x \to a}\left(\frac{1/\sqrt{x} - 1/\sqrt{a}}{x - a}\right)$$

$$= \lim_{x \to a}\left(\frac{1}{x-a}\left[\frac{\sqrt{a} - \sqrt{x}}{\sqrt{x}\sqrt{a}}\right]\right)$$

$$= \lim_{x \to a}\left(\frac{-1}{x-a}\left[\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x}\sqrt{a}}\right]\left[\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}\right]\right)$$

$$= \lim_{x \to a}\left(\frac{-\cancel{(x-a)}}{\cancel{(x-a)}\sqrt{x}\sqrt{a}(\sqrt{x}+\sqrt{a})}\right)$$

$$= \lim_{x \to a}\left(\frac{-1}{\sqrt{x}\sqrt{a}(\sqrt{x}+\sqrt{a})}\right)$$

$$= \boxed{\frac{-1}{2(\sqrt{a})^3} = \text{Slope of tangent line to } y = 1/\sqrt{x} \text{ at } (a, 1/\sqrt{a})}$$

(b.) Using the point slope formula twice we obtain,

$$y = f(a) + f'(a)(x - a)$$

for $(1,1)$: $y = 1 + f'(1)(x-1) = \boxed{1 - \frac{1}{2}(x-1) = y}$

for $(4, 1/2)$: $y = 4 + f'(4)(x-4) = \boxed{4 - \frac{1}{16}(x-4) = y}$

__Remark__: You could also approach this from the other popular notation,

$$m = \lim_{h \to 0}\left(\frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h}\right)$$

its of the same difficulty.

$\boxed{\S 3.1 \# 14}$ Let $H(t) = 10t - 1.86t^2$ be position above surface of Mars where the surface is at $H = 0$.

(a.) $V(1) = \lim_{h \to 0} \left[\frac{10(1+h) - 1.86(1+h)^2 - (10 - 1.86)}{h} \right]$

$= \lim_{h \to 0} \left[\frac{\cancel{10} + 10h - \cancel{1.86} - 3.72h - 1.86h^2 - \cancel{10} + \cancel{1.86}}{h} \right]$

$= \lim_{h \to 0} \left[10 - 3.72 - 1.86\underbrace{h}_{0} \right] = 10 - 3.72 = \boxed{6.28 \frac{m}{s}}$

(b.) $V(a) = \lim_{h \to 0} \left[\frac{10(a+h) - 1.86(a+h)^2 - (10a - 1.86a^2)}{h} \right]$

$= \lim_{h \to 0} \left[\frac{\cancel{10a} + 10h - \cancel{1.86a^2} - 3.72ah - 1.86h^2 - \cancel{10a} + \cancel{1.86a^2}}{h} \right]$

$= \lim_{h \to 0} \left[10 - 3.72a - 1.86\underbrace{h}_{0} \right] = \boxed{10 - 3.72a = V(a)}$

(c.) $H = 0 = 10t - 1.86t^2$

$t(10 - 1.86t) = 0 \therefore \underbrace{t_1 = 0}_{\text{begin}} \ \& \ \underbrace{t_2 = \frac{10}{1.86}}_{\text{end}} \approx 5.38$

$\therefore \boxed{t_2 = 5.38s}$

(d.) $V(t_2) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 2\cancel{(1.86)}\frac{10}{\cancel{1.86}} = 10 - 20 = -10$

$\boxed{V_{\text{hits ground}} = -10 \, m/s}$

$\left[\begin{array}{c} \text{no friction} \\ E_i = E_f \\ KE_i = KE_f \\ \text{since } PE_i = PE_f \end{array} \right]$

digression, don't worry.