Math 126, in-class exercise 4 on limits: Name: ______________________.

1.) Calculate the following indeterminant limit:

$$\lim_{h\to 0}\left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right) = \lim_{h\to 0}\left[\frac{\sqrt{x+h}-\sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)\right]$$

$$= \lim_{h\to 0}\left[\frac{(\sqrt{x+h})^2-\sqrt{x}\sqrt{x+h}+\sqrt{x+h}\sqrt{x}-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}\right]$$

$$= \lim_{h\to 0}\left[\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}\right]$$

$$= \lim_{h\to 0}\left[\frac{1}{\sqrt{x+h}+\sqrt{x}}\right]$$

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$= \boxed{\frac{1}{2\sqrt{x}}.}$$

2.) For the piecewise-defined function $f(x) = \begin{cases} x^2+2 & if\ x<2 \\ \frac{A(2x-4)}{x^2+x-6} & if\ x\geq 2 \end{cases}$ find the value for the constant A which makes the function continuous everywhere.

Clearly $\lim_{x\to a} f(x) = f(a)$ for $a \neq 2$ so f is continuous at all points except possibly 2. At $a=2$ we require $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$ for continuity

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^2+2) = 4+2 = 6.$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}\left(\frac{A(2x-4)}{x^2+x-6}\right)$$

$$= \lim_{x\to 2^+}\left(\frac{2A\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}\right)$$

$$= \frac{2A}{5}$$

Thus, $\frac{2A}{5} = 6 \Rightarrow \boxed{A = 15.}$