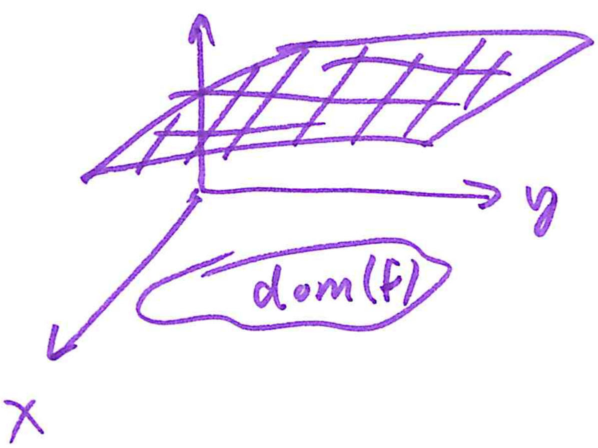
Lecture 15: Partial Differentiation with Three or More Variables & Applications

(pg. 170 – 179 in lecture notes 2020)

$$z = \overbrace{f(x,y) = C}^{\text{level curves}} \qquad\qquad w = \overbrace{f(x,y,z) = C}^{\text{level surface}}$$

$$x^2+y^2+z^2 = C$$



$$\frac{\partial x_i}{\partial x_j} = \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases}$$

$$\boxed{\begin{array}{lll} \dfrac{\partial x}{\partial x} = 1 & \dfrac{\partial x}{\partial y} = 0 & \dfrac{\partial x}{\partial z} = 0 \\[2ex] \dfrac{\partial y}{\partial x} = 0 & \dfrac{\partial y}{\partial y} = 1 & \dfrac{\partial y}{\partial z} = 0 \\[2ex] \dfrac{\partial z}{\partial x} = 0 & \dfrac{\partial z}{\partial y} = 0 & \dfrac{\partial z}{\partial z} = 1 \end{array}} \leftarrow 1 \le i,j \le 3.$$

$$f(x_1, x_2, \dots, x_n) \rightsquigarrow \frac{\partial x_i}{\partial x_j} = \delta_{ij}$$

$D_{\hat{u}} f(P) = \text{const}$

$\hat{u} = \langle a,b \rangle$

$n=2$

eq^n for a, b

$a^2+b^2=1$

$n=3$ $\quad \hat{u} = \langle a,b,c \rangle$

eq^n for $(a,b)\,c$

$a^2+b^2+c^2=1$