$$= \nabla f \cdot \frac{d\vec{r}}{dt}$$

Likewise,

$$\frac{d}{dt}\Big[f(x_1(t), \dots, x_n(t))\Big] = \frac{\partial f}{\partial x_1}\frac{dx_1}{dt} + \dots + \frac{\partial f}{\partial x_n}\frac{dx_n}{dt} = \nabla f \cdot \frac{d\vec{r}}{dt}$$

<u>Remark</u>: If $t \mapsto \vec{r}(t)$ parametrizes a curve on $F(x_1, x_2, \dots, x_n) = 0$ then $F(\vec{r}(t)) = 0$. Then

$$\frac{d}{dt}\Big[F(\vec{r}(t))\Big] = \underbrace{\nabla F(\vec{r}(t))}_{\perp \text{ to the curve}} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{\text{defines the direction of the path.}} = 0$$

<u>Example</u>:

$$\frac{d}{dt}\Big[f(x(t), y(t), z(t), w(t))\Big] = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial w}\frac{dw}{dt}$$

Def/ PDE = [illegible]

ODE = Ordinary Diff. Eq^{n} (one indep. variable)