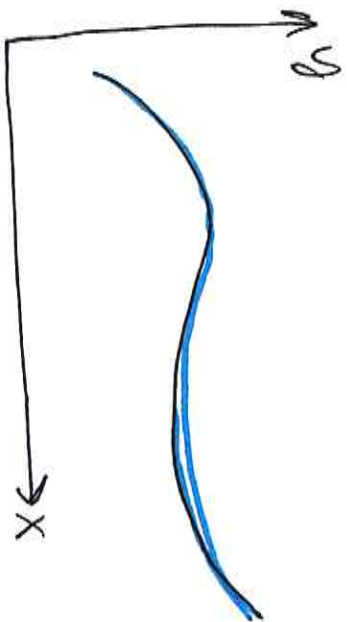


CURVES AND PATHS

①



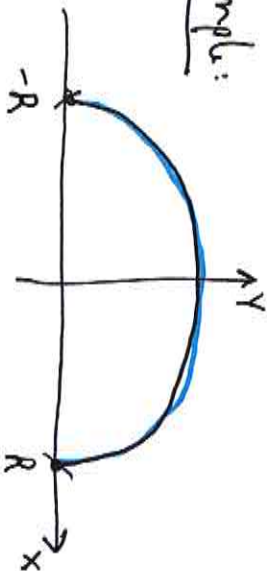
$n=2$

graph: $y = f(x)$

level curve: $F(x, y) = C$

path or parametrization: $\vec{r}(t) = \langle x(t), y(t) \rangle$
is oriented

Example:



graph: $y = \sqrt{x^2 - R^2}$

level curve: $x^2 + y^2 = R^2$ // $F(x, y) = x^2 + y^2$
with $y \geq 0$

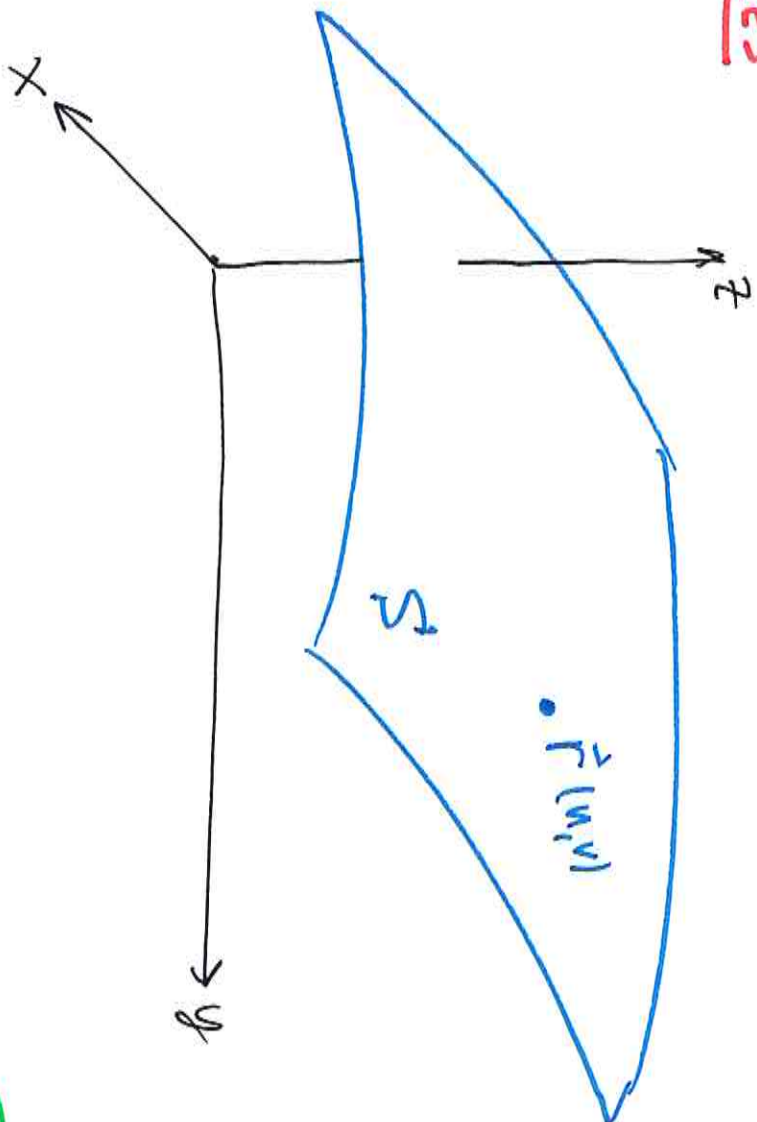
path:

$x = R \cos t$ // $\vec{r}(t) = \langle R \cos t, R \sin t \rangle$
 $y = R \sin t$ // $t \in [0, \pi]$
 $0 \leq t \leq \pi$



Def: / $\vec{r}(t)$ parametrized level curve
 $C = F^{-1}\{k\} = \{(x, y) \in \mathbb{R}^2 \mid F(x, y) = k\}$
iff $F(\vec{r}(t)) = k$ for all $t \in \text{dom}(\vec{r})$.

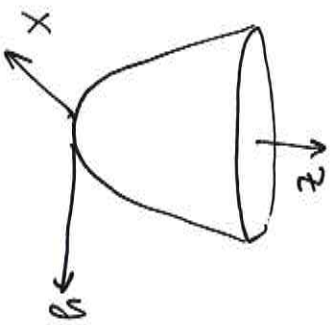
SURFACES



GRAPH

$$z = f(x, y)$$

$$z = x^2 + y^2$$



LEVEL SURFACE

$$F(x, y, z) = C$$

$$x^2 + y^2 - z = 0$$

$$F(x, y, z)$$

PARAMETRIZED SURFACE

aka "PATCH"

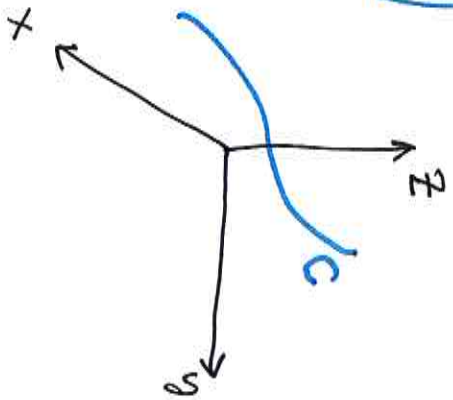
$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

$$\vec{r}_1(x, y) = \langle x, y, x^2 + y^2 \rangle$$

$$\vec{r}_2(\theta, z) = \langle \underbrace{\sqrt{z} \cos \theta}_x, \underbrace{\sqrt{z} \sin \theta}_y, z \rangle$$

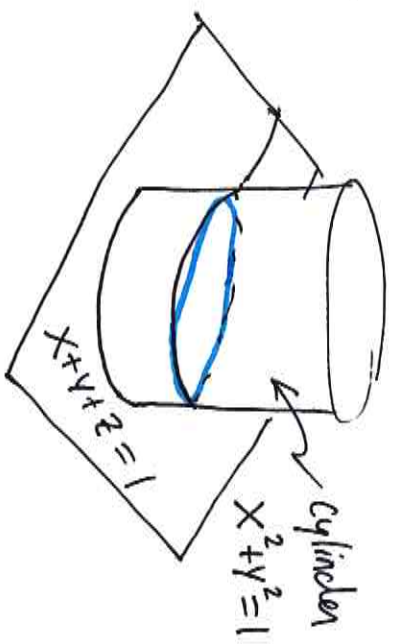
$$x^2 + y^2 = (\sqrt{z})^2 (\cos^2 \theta + \sin^2 \theta) = z$$

$n=3$



$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$t \longmapsto \vec{r}(t) = \langle x_1(t), \dots, x_n(t) \rangle$$



②

$$\begin{aligned} X &= \cos t \\ Y &= \sin t \end{aligned}$$

$$Z = 1 - X - Y = 1 - \cos t - \sin t$$

$$\vec{r}(t) = \langle \cos t, \sin t, 1 - \cos t - \sin t \rangle$$