§4.5 # 6–18 p. 305	find the limits below. Use L'Hopital's Rule where appropriate, indicate why the rule applied by mentioning type $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$

6.) $$\lim_{x\to 1}\left(\frac{x^a-1}{x^b-1}\right) \underset{\left(\frac{0}{0}\right)}{\overset{\ell}{=}} \lim_{x\to 1}\left(\frac{a x^{a-1}}{b x^{b-1}}\right) = \boxed{\frac{a}{b}}$$

8.) $$\lim_{x\to 0}\left(\frac{x+\tan(x)}{\sin(x)}\right) \underset{\left(\frac{0}{0}\right)}{\overset{\ell}{=}} \lim_{x\to 0}\left(\frac{1+\sec^2(x)}{\cos(x)}\right) = \frac{1+\sec^2(0)}{\cos(0)} = \boxed{2}$$

10.) $$\lim_{x\to \pi}\left(\frac{\tan(x)}{x}\right) = \frac{\tan(\pi)}{\pi} = \boxed{0}$$ (Not indeterminant to begin with, no need for L'Hop.)

12.) $$\lim_{x\to \infty}\left(\frac{\ln(\ln(x))}{x}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\ell}{=}} \lim_{x\to \infty}\left(\frac{\frac{1}{\ln(x)}\cdot\frac{1}{x}}{1}\right) = \lim_{x\to\infty}\left(\frac{1}{x\ln(x)}\right) = \boxed{0}$$

14.) $$\lim_{x\to \infty}\left(\frac{e^x}{x^3}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\ell}{=}} \lim_{x\to \infty}\left(\frac{e^x}{3x^2}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\ell}{=}} \lim_{x\to \infty}\left(\frac{e^x}{6x}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\ell}{=}} \lim_{x\to \infty}\left(\frac{e^x}{6}\right) = \boxed{\infty}$$

16.) $$\lim_{x\to 0}\left(\frac{\cos(mx)-\cos(nx)}{x^2}\right) \underset{\left(\frac{0}{0}\right)}{\overset{\ell}{=}} \lim_{x\to 0}\left(\frac{-m\sin(mx)+n\sin(nx)}{2x}\right)$$

$$\underset{\left(\frac{0}{0}\right)}{\overset{\ell}{=}} \lim_{x\to 0}\left(\frac{-m^2\cos(mx)+n^2\cos(nx)}{2}\right)$$

$$= \boxed{\frac{n^2-m^2}{2}}$$ {remember $\cos(0)=1$.}

18.) $$\lim_{x\to 0}\left(\frac{x}{\tan^{-1}(4x)}\right) \underset{\left(\frac{0}{0}\right)}{\overset{\ell}{=}} \lim_{x\to 0}\left(\frac{1}{\frac{1}{1+(4x)^2}\cdot\frac{d}{dx}(4x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{1+16x^2}{4}\right)$$

$$= \boxed{\frac{1}{4}}$$

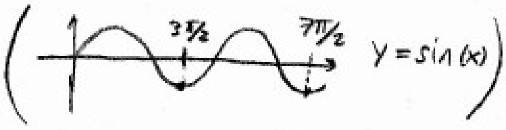
§4.5 #20–28 p. 305	Find the limits

20.) $\lim\limits_{x\to 0}\left(\dfrac{1-e^{-2x}}{\sec(x)}\right) = \dfrac{1-e^{0}}{1} = \dfrac{0}{1} = \boxed{0}$ ← why no $\mathcal{L}$'Hopt. ?

22.) $\lim\limits_{x\to -\infty}\left(x^2e^x\right) = \lim\limits_{x\to -\infty}\left(\dfrac{x^2}{e^{-x}}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to -\infty}\left(\dfrac{2x}{-e^{-x}}\right) \underset{\left(\frac{\infty}{\infty}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to -\infty}\left(\dfrac{2}{e^{-x}}\right) = \boxed{0}$

24.) $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}\left(\sec(7x)\cos(3x)\right) = \lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}\left(\dfrac{\cos(3x)}{\cos(7x)}\right) \underset{\left(\frac{0}{0}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}\left(\dfrac{-3\sin(3x)}{-7\sin(7x)}\right) = \boxed{\dfrac{3}{7}}$

$\underset{\left(\frac{0}{0}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}\left(\dfrac{-3\sin(3x)}{-7\sin(7x)}\right)$ (recall sin



$= \dfrac{3}{7}\,\dfrac{\cancel{\sin\left(\frac{3\pi}{2}\right)}\to -1}{\cancel{\sin\left(\frac{7\pi}{2}\right)}\to -1}$

$= \boxed{\dfrac{3}{7}}$

26.) $\lim\limits_{x\to 1^+}\left((x-1)\tan\left(\frac{\pi x}{2}\right)\right) = \lim\limits_{x\to 1^+}\left(\dfrac{x-1}{\cot\left(\frac{\pi x}{2}\right)}\right)$: $\cot(\theta) = \dfrac{\cos\theta}{\sin\theta}$

$\underset{\left(\frac{0}{0}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to 1^+}\left(\dfrac{1}{-\frac{\pi}{2}\csc^2\left(\frac{\pi x}{2}\right)}\right)$

$\boxed{\csc(x) = \dfrac{1}{\sin(x)} \quad \csc\left(\frac{\pi}{2}\right) = \dfrac{1}{\sin(\pi/2)}}$

$= \boxed{\dfrac{-2}{\pi}}$

28.) $\lim\limits_{x\to 0}\left(\csc(x) - \cot(x)\right) = \lim\limits_{x\to 0}\left(\dfrac{1}{\sin(x)} - \dfrac{\cos(x)}{\sin(x)}\right)$

$= \lim\limits_{x\to 0}\left(\dfrac{1-\cos(x)}{\sin(x)}\right)$

$\underset{\left(\frac{0}{0}\right)}{\overset{\mathcal{L}}{=}} \lim\limits_{x\to 0}\left(\dfrac{\sin(x)}{\cos(x)}\right)$

$= \dfrac{\sin(0)}{\cos(0)} = \boxed{0}$

§4.5 # 30–36 p. 305	Find the limit

30.) $\lim\limits_{x\to 1}\left(\frac{1}{\ln(x)}-\frac{1}{x-1}\right) = \lim\limits_{x\to 1}\left(\frac{x-1-\ln(x)}{(x-1)\ln(x)}\right)$

$\overset{\ell}{\underset{(\frac{0}{0})}{=}} \lim\limits_{x\to 1}\left(\frac{1-\frac{1}{x}}{\ln(x)+(x-1)\frac{1}{x}}\right)$ $\quad (x-1)\frac{1}{x} = 1-\frac{1}{x}$

$\overset{\ell}{\underset{(\frac{0}{0})}{=}} \lim\limits_{x\to 1}\left(\frac{\frac{-1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}\right) = \boxed{\frac{-1}{2}}$

32.) $\lim\limits_{x\to 0^+}(\sin(x))^{\tan(x)} = e^{\underbrace{\lim_{x\to 0^+}\left(\tan(x)\ln(\sin(x))\right)}_{(*)}}$ $\quad$ using pg. 63 of notes.

$(*)$: $\lim\limits_{x\to 0^+}\left(\frac{\ln(\sin(x))}{\cot(x)}\right) \overset{\ell}{\underset{(\frac{\infty}{\infty})}{=}} \lim\limits_{x\to 0^+}\left(\frac{\frac{\cos(x)}{\sin(x)}}{-\csc^2(x)}\right) = \lim\limits_{x\to 0^+}\left(\sin(x)\cos(x)\right) = 0$

Thus $\lim\limits_{x\to 0^+}(\sin(x))^{\tan(x)} = e^0 = \boxed{1}$

34.) See pg. 64 of Lecture notes $\boxed{E11}$ is basically this problem.

36.) $\lim\limits_{x\to\infty}\left(x^{\frac{\ln(2)}{1+\ln(x)}}\right) = e^{\lim_{x\to\infty}\left[\ln\left(x^{\frac{\ln(2)}{1+\ln(x)}}\right)\right]}$

$= e^{\underbrace{\lim_{x\to\infty}\left[\frac{\ln(2)}{1+\ln(x)}\cdot\ln(x)\right]}_{(*)}} \underset{\text{(using * below)}}{=} e^{\ln(2)} = \boxed{2}$

$*$: $\lim\limits_{x\to\infty}\left(\frac{\ln(2)\ln(x)}{1+\ln(x)}\right) \overset{\ell}{\underset{(\frac{\infty}{\infty})}{=}} \lim\limits_{x\to\infty}\left(\frac{\frac{\ln(2)}{\cancel{x}}}{\frac{1}{\cancel{x}}}\right) = \underline{\ln(2)}$

52.)

$\lim\limits_{x\to\infty}\left(\frac{\ln(x)}{x^P}\right) \overset{\ell}{\underset{(\frac{\infty}{\infty})}{=}} \lim\limits_{x\to\infty}\left(\frac{\frac{1}{x}}{Px^{P-1}}\right)$ $\quad$ (assume $P>0$ as in text.)

$= \lim\limits_{x\to\infty}\left(\frac{1}{Px^P}\right)$

$= 0$

- Thus the power function x^P for $P>0$ grows faster than the natural log at ∞.