Section 2.1 Set Concepts — Week 1

I. Definition

- A set is a collection of objects, which are called elements or members of the set.

- A set is well-defined if its contents can be clearly determined.

II. Notation

Three methods are commonly used to indicate a set:

(1) description

(2) roster form

(3) set-builder notation

III A Examples using description

(1) Elements: Matthew, Mark, Luke, John, Acts

- Using description: The set is the first five books in the New Testament.

(2) Elements: penny, nickel, dime, quarter, dollar

- The set is the coins in the U.S.

(3) Elements: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday.

- The set is the days of the week.

III.B. Examples using roster form

(1) Elements: a, b, c, d, e

Set $A = \{a, b, c, d, e\}$ we list the elements inside a pair of braces, $\{\ \}$.

(2) Elements: 1, 2, 3, 4, 5

Set $A = \{1, 2, 3, 4, 5\}$

(3) Natural number $\mathbb{N}$

$N = \{1, 2, 3, 4, 5, \ldots\}$

$\cdots$, called ellipsis, indicate that the elements in the set continue in the same manner.

Examples of ellipsis

(1) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$= \{1, 2, 3, \ldots, 10\}$

(2) $B = \{2, 4, 6, 8, 10, 12, \ldots\}$ = set of all positive even numbers.

III.C. Examples using set-builder notation

D	=	{	x	|	condition(s) }
↑	↑	↑	↑	↑	↑
set D	is	the set of	all element x	such that	the condition(s) x must meet in order to be a member of the set

- The symbol ∈, "the element of" is used to indicate membership.

(1) $D = \{1, 2, 3, 4, 5\}$
$= \{x \mid x \in \mathbb{N} \text{ and } x \leq 5\}$

(2) $B = \{10, 11, 12, 13, 14, \ldots\}$
$= \{x \mid x \in \mathbb{N} \text{ and } x \geq 10\}$

(3) $C = \{2, 4, 6, 8, 10, 12 \ldots\}$
$= \{x \mid x = 2m \text{ and } m \in \mathbb{N}\}$

IV. Definition

- A set is said to be finite if it either contains no element or the number of elements in the set is a natural number.

- A set that is not finite is said to be infinite.

- Set A is equal to set B, symbolized by $A = B$, if and only if set A and set B contain exactly the same elements.

Example: $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 1, 4\}$
$A = B$, order is not important.

- The <u>cardinal number</u> of set A, symbolized by $n(A)$, is the number of elements of set A.

Example : $A = \{a, b, c, d, e\}$; $n(A) = 5$

$B = \{1, 2, 3, \ldots, 10\}$: $n(B) = 10$.

- Set A is <u>equivalent</u> to set B if and only if $n(A) = n(B)$.

Example : $A = \{\alpha, \beta, \gamma\}$; $B = \{a, b, c\}$

$n(A) = 3$, $n(B) = 3$

$\therefore$ set A is equivalent to set B.

- Two sets that are equivalent can be placed in <u>one-to-one correspondence</u>.

Example : $G = \{A, B, C, D\}$

$S = \{80\text{-}89,\ 60\text{-}69,\ 70\text{-}79,\ 90\text{-}100\}$

- The set that contains <u>no elements</u> is called the <u>empty set</u> or null set and is symbolized by $\{\ \}$ or ϕ.

Caution : $\{\emptyset\}$ and $\{0\}$ are not empty!

- <u>A universal set</u>, symbolized by U, is a set that contains all the elements for any specific discussion.

Example : $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then only the natural numbers 1 through 10 may be used in the problem.

I. Definition (Subset)

Set A is a subset of set B, symbolized by $A \subseteq B$, if and only if all the elements of set A are also elements of set B.

II. Example

Determine whether set A is a subset of set B.

(1) $A = \{a, b, c, d\}$

$B = \{a, b, c, d, e\}$

All elements in set A are also in set B, so $A \subseteq B$.

(2) $A = \{1, 3, 5, 7, 9, \ldots\}$

$B = \{2, 4, 6, 8, 10, \ldots\}$

None of the element in set A is in set B, so $A \nsubseteq B$.

(3) A = {water, pepsi, coke}

B = {sweet tea, water, milk}

Pepsi $\in A$ but pepsi $\notin B$, so $A \nsubseteq B$.

III. Definition (Proper subset)

Set A is a proper subset of set B, symbolized by $A \subset B$, if and only if all the elements of set A are elements of set B and set A $\neq$ set B (ie set B must contain at least one element not in set A)

IV. Example.

Determine whether set A is a proper subset of set B.

(1) $A = \{\alpha, \beta, \gamma\}$

$B = \{\alpha, \gamma, \beta, \sigma\}$

Since all elements in A are also in B and $A \neq B$, $A \subset B$.

(2) $A = \{1, 2, 3, 4\}$, $B = \{4, 2, 1, 3\}$

Since $A = B$, $A \not\subset B$.

N.B. 1) Every set is a subset of itself, but no set is a proper subset of itself.
2) The empty set is a subset of every set, including itself.

V. Number of subsets

The number of distinct subsets of a finite set A is 2^n, where n is the number of element in set A.

Example: List all the distinct subsets for the set $\{M, A, T, H\}$

$\{M, A, T, H\}$	$\{M, A, T\}$	$\{M, A\}$	$\{M\}$	$\{\,\}$
	$\{M, A, H\}$	$\{M, T\}$	$\{A\}$	
	$\{A, T, H\}$	$\{M, H\}$	$\{T\}$	
	$\{M, T, H\}$	$\{A, T\}$	$\{H\}$	
		$\{A, H\}$		
		$\{T, H\}$		

$1 + 4 + 6 + 4 + 1 = 16 = 2^4$