Section 3.2 Week 3.

Defn: A truth table is a device used to determine when a compound statement is true or false.

I. Negation

Recall that if p is a true statement, then "not p" is a false statement.
if p is a false statement, then "not p" is a true statement.

Negation

	p	$\sim p$
case 1	T	F
case 2	F	T

II. Conjunction

To illustrate the conjunction, consider the following situation:
The salesperson promises that carpet will be delivered on Thursday and furniture will be delivered on Friday.

Let p: carpet will be delivered on Thursday
q: furniture will be delivered on Friday.

	p	q	$p \wedge q$	
Case 1	T	T	T	• carpet delivered on Thu and furniture delivered on Fri
Case 2	T	F	F	• carpet delivered on Thu but furniture not delivered on Fri.
Case 3	F	T	F	• carpet not delivered on Thu. but furniture delivered on Fri
Case 4	F	F	F	• carpet not delivered on Thu and furniture not delivered on Fri

the statement $p \wedge q$ is true only when both p & q are true

Example: construct a truth table for $p \wedge \sim q$

p	q	p	$\wedge$	$\sim$	q
T	T	T	F	F	T
T	F	T	T	T	F
F	T	F	F	F	T
F	F	F	F	T	F
		1	4	3	2

Example: construct a truth table for the statement:
It is false that Wanda Garner is the president and that Judy Ackerman is the treasurer.

Let p: Wanda Garner is the president.
q: Judy Ackerman is the treasurer.
$\sim(p \wedge q)$: It is false that Wanda Garner is the president and Judy A is the treasurer

p	q	$\sim$	$(p$	$\wedge$	$q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	T	F	F	T
F	F	T	F	F	F
		4	1	3	2

a) Under what condition is the compound statement false?
When both p & q are true.

III. Disjunction

To illustrate disjunction, consider the following situation
Requirement for a job is a two-year college degree or 5 years of related experience.

Let p: 2-year college degree
q: 5 years of related experience.

p	q	$p \vee q$	
T	T	T	qualified
T	F	T	qualified
F	T	T	qualified
F	F	F	not qualified

The disjunction, $p \vee q$ is false only when both p & q are false

Example: construct the truth table for $\sim(p \vee \sim q)$

p	q	$\sim$	$(p$	$\vee$	$\sim q)$
T	T	F	T	T	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	F	F	T	T
		4	1	3	2

the statement $\sim(p \vee \sim q)$ is true only when p is F & q is T.

Example: construct the truth table for $(p \wedge \sim q) \vee r$

p	q	r	$(p$	$\wedge$	$\sim q)$	$\vee$	r	
T	T	T	T	F	F	T	T	T
T	T	F	T	F	F	F	F	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	F	T
F	T	T	F	F	F	T	T	T
F	T	F	F	F	F	F	F	F
F	F	T	F	F	T	T	T	T
F	F	F	F	F	T	F	F	F
			1	3	2	5	4	

Example: suppose you know the truth value of a compound statement for a specific case, we can do the following instead of working out the whole truth table.

p is F, q is T, r is T

$(p \wedge \sim q) \vee r$

$(F \wedge F) \vee T$

$F \vee T$

T.

Section 3.3 Week 3

I. Conditional

To see what the truth table for a conditional statement is, consider:

p: You get an A

q: I buy you a car.

$p \to q$: If you get an A, then I buy you a car.

Assume this statement is true unless I broke my promise (ie buy you a car)

p	q	p	→	q
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F
		1	3	2

- you get an A & I buy you a car
- you get an A & I do not buy you a car
- you don't get an A & I buy you a car
- you don't get an A & I don't buy you a car

The conditional statement $p \to q$ is true in every case except when p is T but q is false.

<u>Example</u>: Construct the truth table for $(\sim p \vee q) \to \sim r$

p	q	r	($\sim$p	∨	q)	⟶	$\sim$r	
T	T	T	F	T	T	F	F	F
T	T	F	F	T	T	T	T	T
T	F	T	F	F	F	T	F	T
T	F	F	F	F	F	T	T	T
F	T	T	T	T	T	F	F	F
F	T	F	T	T	T	T	T	T
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	T	T	T
			1	3	2	5	4	

II. Biconditional

The biconditional statement, $p \leftrightarrow q$, means $p \to q$ and $q \to p$, ie $(p \to q) \wedge (q \to p)$. To determine the truth table for $p \leftrightarrow q$, we develop the one for $(p \to q) \wedge (q \to p)$:

p	q	$(p$	$\to$	$q)$	$\wedge$	$(q$	$\to$	$p)$
T	T	T	T	T	T	T	T	T
T	F	T	F	F	F	F	T	T
F	T	F	T	T	F	T	F	F
F	F	F	T	F	T	F	T	F
		1	3	2	7	4	6	5

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

The biconditional statement $p \leftrightarrow q$, is true only when p and q have the same truth value, that is, when both are truth or both are false.

Example: construct a truth table for $p \leftrightarrow (q \to \sim r)$

p	q	r	p	$\leftrightarrow$	$(q$	$\to$	$\sim r)$
T	T	T	T	F	T	F	F
T	T	F	T	T	T	T	T
T	F	T	T	T	F	T	F
T	F	F	T	T	F	T	T
F	T	T	F	T	T	F	F
F	T	F	F	F	T	T	T
F	F	T	F	F	F	T	F
F	F	F	F	F	F	T	T
			1	5	2	4	3

III. Specific cases

To find the truth value of a compound statement for a specific case, we can do the following:

Ex: p is F, q is T, r is T

$$(\sim p \leftrightarrow q) \rightarrow (\sim q \leftrightarrow r)$$
$$(T \leftrightarrow T) \rightarrow (F \leftrightarrow T)$$
$$T \rightarrow F$$
$$F$$

Ex: p is T, q is F, r is F

$$(\sim p \rightarrow q) \leftrightarrow (\sim p \rightarrow r)$$
$$(F \rightarrow F) \leftrightarrow (F \rightarrow F)$$
$$T \leftrightarrow T$$
$$T$$

IV. self-contradictions, Tautologies, and Implications

Defn:
- A self-contradiction is a compound statement that is always false
- A tautology is a compound statement that is always true.
- An implication is a conditional statement that is a tautology.

Ex: 1) $p \wedge (q \wedge \sim p)$

p	q	p	$\wedge$	$(q$	$\wedge$	$\sim p)$
T	T	T	F	T	F	F
T	F	T	F	F	F	F
F	T	F	F	T	T	T
F	F	F	F	F	F	T
		1	5	2	4	3

It is a self-contradiction since the statement is always false

2) $(\sim q \rightarrow p) \vee \sim q$

p	q	$(\sim q$	$\rightarrow$	$p)$	$\vee$	$\sim q$
T	T	F	T	T	T	F
T	F	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	F	F	T	T
		1	4	2	5	3

Since the statement is always true, it is a tautology.

3) $(q \wedge p) \longrightarrow (p \wedge q)$

p	q	$(q$	$\wedge$	$p)$	$\rightarrow$	$(p$	$\wedge$	$q)$
T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F	F
F	T	T	F	F	T	F	F	T
F	F	F	F	F	T	F	F	F
		1	3	2	7	4	6	5

Since the conditional statement is always true (ie a tautology), it is an implication.