*Please print this out and write your solutions on this document. 30pts to earn here. Thanks!*

**Problem 11:** (4pts) Solve the inequalities below and express your answer in interval notation as well as with a number line picture.

(a.) $5 - 3x \leq 8x - 7$,

$5 + 7 \leq 8x + 3x$

$12 \leq 11x \Rightarrow x \geq \frac{12}{11} \Rightarrow \boxed{[12/11, \infty)}$


(b.) $\frac{2}{3} - \frac{1}{2}x \geq \frac{1}{6} + x$,

$4 - 3x \geq 1 + 6x$

$3 \geq 9x \Rightarrow x \leq \frac{1}{3} \Rightarrow \boxed{(-\infty, 1/3]}$

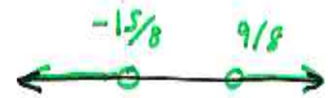

(c.) $|8x + 3| > 12$,

$8x + 3 < -12$ or $8x + 3 > 12$

$8x < -15$ or $8x > 9$

$x < -15/8$ or $x > 9/8$

$\boxed{(-\infty, -15/8) \cup (9/8, \infty)}$




(d.) $|7x - 14| + 3 \leq 10$.

$|7x - 14| \leq 7$

$-7 \leq 7x - 14 \leq 7$

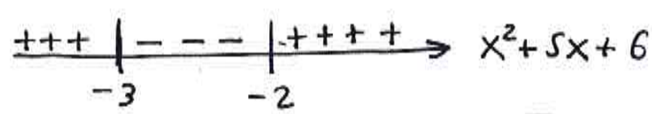
$7 \leq 7x \leq 21$

$1 \leq x \leq 3 \rightarrow \boxed{[1, 3]}$


**Problem 12:** (7pts) Solve the following inequalities using an appropriate technique. Show your work and write the answer using interval notation ( you might need to use $\cup$ for union )

(a.) $\underbrace{x^2 + 5x + 6}_{(x+2)(x+3)} \geq 0$,

$(x + 2)(x + 3) \geq 0$




$\therefore \boxed{(-\infty, -3] \cup [-2, \infty)}$

(b.) $\frac{1}{x^2 + 5x + 6} < 0$,

same sign-chart as part (a). However, must exclude $x = -2, -3$ because of division by zero,

$\boxed{(-3, -2)}$

Also this is < so we use the - - - part to form answer.

(c.) $\dfrac{x^2(x+3)}{x^2-13x+12} \geq 0,$

$$\frac{x^2(x+3)}{(x-12)(x-1)} \geq 0$$

algebraic critical #'s are $0, -3, 1, 12$

$$\underset{-3}{---|} \; \underset{0}{+++|} \; \underset{1}{+++|} \; \underset{12}{---|} \; +++ \rightarrow \quad \frac{x^2(x+3)}{x^2-13x+12}$$

$\Rightarrow [-3, 0] \cup [0, 1) \cup (12, \infty)$

$\therefore \boxed{[-3, 1) \cup (12, \infty)}$

(d.) $\dfrac{x^2+3x}{2x^2+8x+10} \geq 0,$

$$f(x) = \frac{x(x+3)}{2(x^2+4x+5)} = \frac{x(x+3)}{2[\underbrace{(x+2)^2+1}_{\text{never } 0 \text{ for } x \in \mathbb{R}}]} \Rightarrow \text{algebraic critical \#'s } 0, -3$$

$$\underset{-3}{+++|} \; \underset{0}{---|} \; +++ \rightarrow f(x) \qquad \therefore \boxed{(-\infty, -3] \cup [0, \infty)}$$

(e.) $x^3 < 4x^2 - 3x,$

$$x^3 - 4x^2 + 3x < 0$$

$$x(x^2 - 4x + 3) < 0$$

$$f(x) = x(x-1)(x-3) < 0$$

algebraic critical #'s are $x = 0, 1, 3$

$$\underset{0}{---|} \; \underset{1}{+++|} \; \underset{3}{---|} \; +++ \rightarrow f(x)$$

$\Rightarrow \boxed{(-\infty, 0) \cup (1, 3)}$

(f.) $\dfrac{3}{x-1} - \dfrac{4}{x} \geq 1,$

$$0 \leq \frac{3}{x-1} - \frac{4}{x} - 1 = \frac{3x - 4(x-1) - 1(x-1)x}{x(x-1)} = \frac{-x^2+4}{x(x-1)}$$

$$\Rightarrow \underbrace{0 \leq \frac{(2-x)(2+x)}{x(x-1)}}_{\text{algebraic critical \#'s are } 0, 1, 2, -2}$$

$$\underset{-2}{---|} \; \underset{0}{+++|} \; \underset{1}{---|} \; \underset{2}{+++|} \; --- \rightarrow \quad \frac{4-x^2}{x(x-1)}$$

$\therefore \boxed{[-2, 0) \cup (1, 2]}$

(g.) $x^4 - 10x^2 + 9 > 0$.

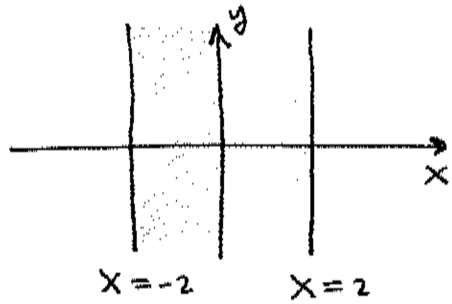
$$(x^2 - 1)(x^2 - 9) > 0$$

$$f(x) = (x+1)(x-1)(x+3)(x-3) > 0$$


$$\boxed{(-\infty, 3) \cup (-1, 1) \cup (3, \infty)}$$

**Problem 13:** (2pts) Plot the following regions:

(a.) $\{(x, y) \mid |x| \leq 2\}$,




$|x| \leq 2$

$-2 \leq x \leq 2$

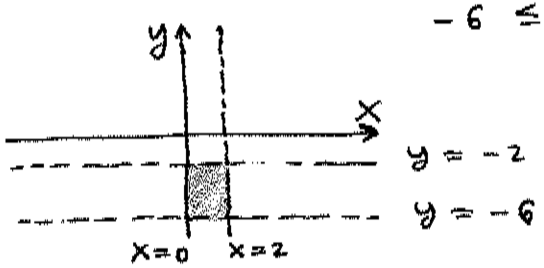
so

(b.) $\{(x, y) \mid \underbrace{|x - 1| \leq 1} \text{ and } \underbrace{|2y + 8| \leq 4}\}$,

$-1 \leq x - 1 \leq 1$ $\qquad$ $-4 \leq 2y + 8 \leq 4$

$0 \leq x \leq 2$ $\qquad$ $-12 \leq 2y \leq -4$

$-6 \leq y \leq -2$




**Problem 14:** (1pt) Given $P_1 = (2, 3)$ and $P_2 = (8, -7)$ find the distance between $P_1$ and $P_2$ and find the location of the midpoint between the given pair of points.

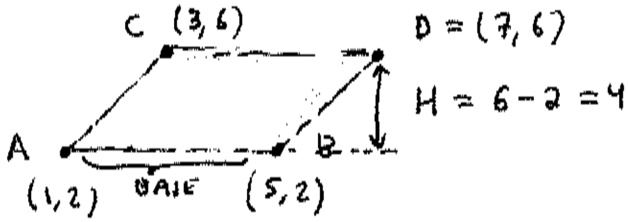
$$d(P_1, P_2) = \sqrt{(8-2)^2 + (-7-3)^2} = \sqrt{36 + 100} = \underbrace{\boxed{\sqrt{136}}}_{\text{distance from } P_1 \text{ to } P_2}$$

$$\text{midpoint} = \frac{P_1 + P_2}{2} = \frac{1}{2}(2 + 8, 3 + (-7))$$

$$= (10/2, -4/2)$$

$$= \underbrace{\boxed{(5, -2)}}_{\text{midpoint of } P_1 \text{ \& } P_2}$$

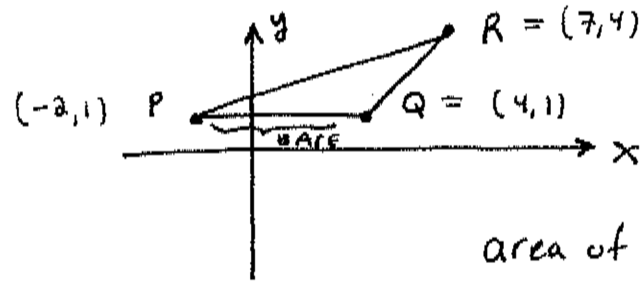
**Problem 15:** (1pt) Given $A = (1,2)$, $B = (5,2)$, $C = (3,6)$ and $D = (7,6)$ form the vertices of a parallelogram $\mathcal{P}$. Find the area of $\mathcal{P}$. Show your work including appropriate diagrams.




BASE = 5 − 1 = 4

area $(\mathcal{P})$ = (BASE)(HEIGHT) = (4)(4) = $\boxed{16.}$

**Problem 16:** (1pt) Find the area of the triangle with vertices $P = (-2,1)$, $Q = (4,1)$ and $R = (7,4)$. Show your work including appropriate diagrams.



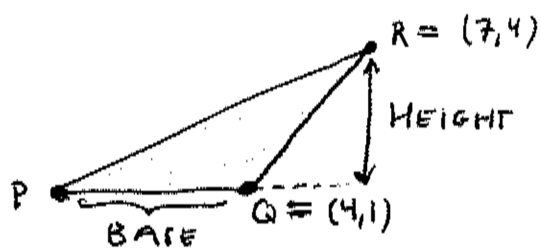

BASE = 4 − (−2) = 6

HEIGHT = 4 − 1 = 3

area of $\Delta$ = $\frac{1}{2}$(BASE)(HEIGHT)

= $\frac{1}{2}(6)(3)$

= $\boxed{9.}$




**Problem 17:** (1pt) Consider the equation $y(x^2+1) = 1$. If $P = (1,1)$, $Q = (1, 1/2)$ and $R = (-1, 1/2)$ then which of the points $P, Q, R$ are on the graph of the given equation ?

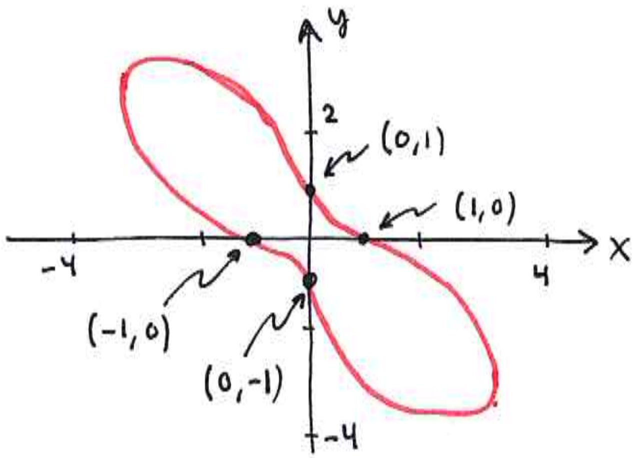
$P = (1,1)$ $y(x^2+1) = 1(1^2+1) = 1(2) = 2 \neq 1$ $\therefore$ P not on graph.

$Q = (1, 1/2)$ $y(x^2+1) = \frac{1}{2}(1^2+1) = \frac{1}{2}(2) \overset{\checkmark}{=} 1$ $\therefore$ Q is on graph of eq$^{\underline{n}}$.

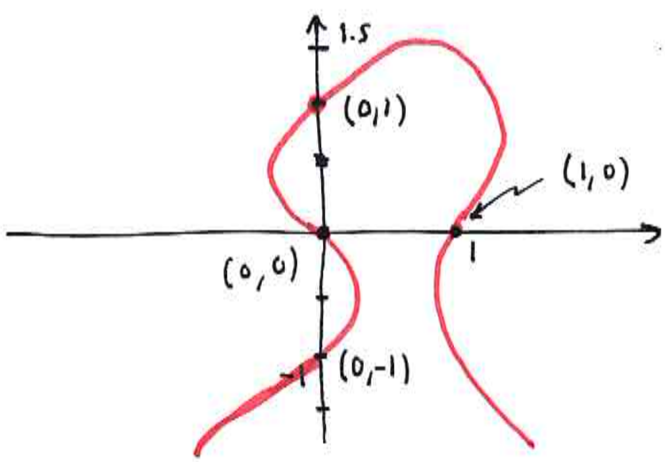
$R = (-1, 1/2)$ $\frac{1}{2}((-1)^2+1) = \frac{1}{2}(2) \overset{\checkmark}{=} 1$ $\therefore$ R is on graph of eq$^{\underline{n}}$ given.

**Problem 18:** (2pt) Use a graphing calculator or appropriate website to plot the following equations. Sketch the result by hand as your answer. Include explicit labels of any $x$ or $y$-intercepts.

(a.) $x^4 + y^4 + 20xy = 1$, (CLICK HERE FOR ANSWER)




(b.) $\dfrac{x+y}{x^4+y^3} = 1$ (CLICK HERE FOR ANSWER)




**Problem 19:** (1pts) The graph of $x^2 - axy + y^2 = 1$ varies according the choice of $a$. Try the linked website graph to explore what happens as we vary $a$ be adjusting the slider in Desmos. Describe the possible graphs in words. (CLICK HERE FOR THE GRAPH)

We see ellipses and hyperbolae tilted into Quadrants II & IV for $a < 0$ and I & III for $a > 0$.

**Problem 20:** (3pts) The standard form of a circle equation is $(x-h)^2+(y-k)^2 = R^2$. In the standard form just given we have a circle with radius $R$ and center $(h,k)$. Use completing the square and algebra as needed to place each circle equation below into standard form. Find the center and radius in each case. (CLICK HERE TO CHECK ANSWER)

(a.) $x^2+y^2+6y+2=0$,

$$x^2 + (y+3)^2 - 9 + 2 = 0$$

$$\boxed{x^2 + (y+3)^2 = 7}$$

Center : $(0,-3)$

$R = \sqrt{7}$

(b.) $x^2+y^2+\frac{1}{2}x+2y+\frac{1}{16}=0$,

$$x^2 + \frac{1}{2}x + y^2 + 2y + \frac{1}{16} = 0$$

$$\left(x+\frac{1}{4}\right)^2 - \cancel{\frac{1}{16}} + (y+1)^2 - 1 + \cancel{\frac{1}{16}} = 0$$

$$\boxed{\left(x+\frac{1}{4}\right)^2 + (y+1)^2 = 1}$$

Center : $(-1/4, -1)$

Radius : $R = 1$

(c.) $3x^2+3y^2+6x-y=0$.

$$x^2 + y^2 + 2x - \frac{1}{3}y = 0$$

$$x^2 + 2x + y^2 - \frac{1}{3}y = 0$$

$$(x+1)^2 - 1 + \left(y-\frac{1}{6}\right)^2 - \frac{1}{36} = 0$$

$$\boxed{(x+1)^2 + \left(y-\frac{1}{6}\right)^2 = \frac{37}{36}}$$

$\therefore$ $(-1, 1/6)$ Center & $R = \sqrt{\frac{37}{36}} = \frac{1}{6}\sqrt{37}$

**Problem 21:** (5pts) Find the equation of a line given that:

**(a.)** the line contains points $(-1,-3)$ and $(0,1)$,

$$m = \frac{\Delta y}{\Delta x} = \frac{1-(-3)}{0-(-1)} = \frac{4}{1} = 4$$

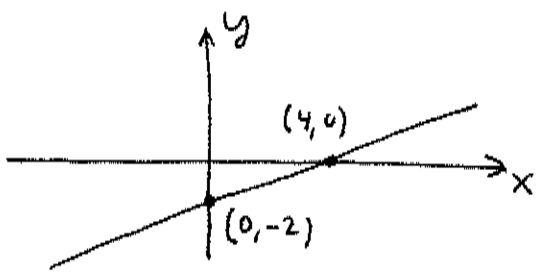
$$y = 1 + 4(x-0) \quad \therefore \quad \boxed{y = 4x+1}$$

Check Answer

$4(-1)+1 = -3$ ✓

$4(0)+1 = 1$ ✓

**(b.)** the line with $x$-intercept 4 and $y$-intercept of $-2$,




$y = mx+b$

we're given $\underline{b=-2}$.

$y = mx - 2$

Then use x-intercept data

$0 = 4m - 2$

$4m = 2 \Rightarrow \underline{m = 1/2}$

$\therefore \boxed{y = \frac{1}{2}x - 2}$

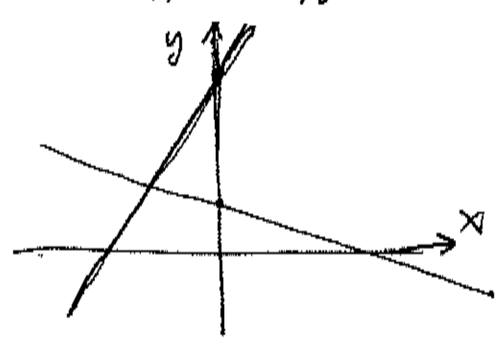
**(c.)** the line perpendicular to $\underbrace{x+3y=2}$ with $y$-intercept 4,

$$y = \frac{2-x}{3} = \frac{-1}{3}x + \frac{2}{3} \quad \text{has } m = \frac{-1}{3}$$

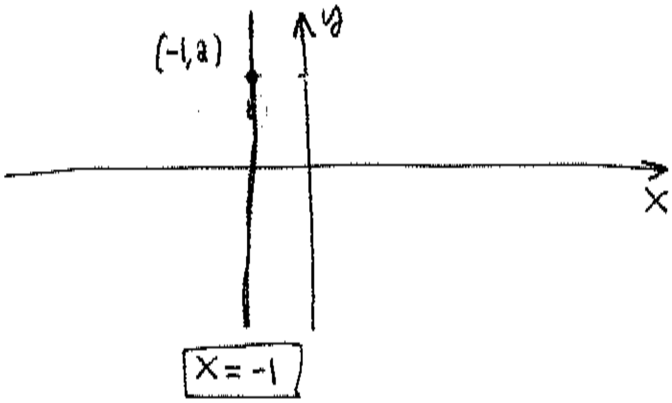
The $\perp$ line has slope $m_\perp = \frac{-1}{m} = \frac{-1}{-1/3} = 3$

and $b_\perp = 4$ thus

$$\boxed{y = 3x + 4}$$

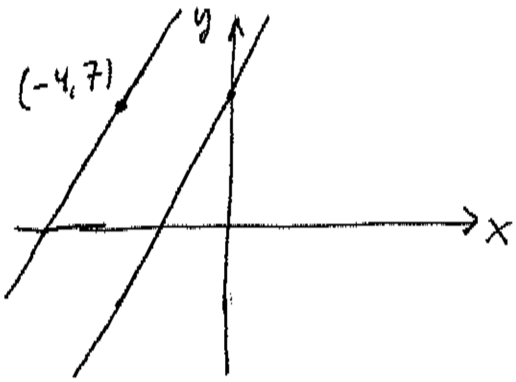


(d.) the line through $(-1, 2)$ which is parallel to the $y$-axis,




$\boxed{X = -1}$

(e.) the line through $(-4, 7)$ which is parallel to the line $y = 3x + 8$.




Same slope as $y = 3X + 8$
hence $m = 3$. Point $(-4, 7)$

$$y = 7 + 3(X - (-4))$$

$$\therefore \boxed{y = 3X + 19}$$

**Problem 22:** (2pts) Use a graphing calculator or appropriate website to solve the following equations graphically. Give answers rounded to two decimals.

(a.) $\underbrace{16x^3 + 16x^2 = x + 1}$ for $-2 \leq x \leq 2$,

Has sol'ns $\boxed{-1, -0.25, 0.25}$

(b.) $1 + \sqrt{x} = \sqrt{1 + x^2}$ for $-1 \leq x \leq 5$. Sorry, bad link ☺

You could use THIS GRAPH to help solve part (a.). Or, another approach for part (b.) is to convert the problem to solving

$$1 + \sqrt{x} - \sqrt{1 + x^2} = 0$$

for $-1 \leq x \leq 5$. Hence look for $x$-intercepts in THIS GRAPH.

Has sol'ns $\boxed{X = 0}$ or $X = \frac{1}{3}\left(2 + \sqrt[3]{53 - 6\sqrt{78}} + \sqrt[3]{53 + 6\sqrt{78}}\right)$

aka $\boxed{X \approx 2.31}$

looked like when we looked in class $X = 2.32$