Please print this out and write your solutions on this document. I will only give half credit if the solutions are not written on this form. Please staple when finished. 60pts to earn here. Thanks!

Problem 24: (2pts) Find the domain of each function and express it using interval notation.

(a.) $g(x) = \frac{\sqrt{2+x}}{3-x}$,

$2+x \geq 0$ and $x \neq 3$

$x \geq -2$ and $x \neq 3$

$\boxed{[-2, 3) \cup (3, \infty)}$

(b.) $f(x) = \frac{(x+1)^2}{\sqrt{2x-1}}$,

$2x-1 > 0 \Rightarrow 2x > 1 \Rightarrow x > 1/2$

$\boxed{(1/2, \infty)}$

Problem 25: (4pts) The difference quotient based at a for $f(x)$ is given by $\frac{f(a)-f(a+h)}{h}$ where $h \neq 0$. Calculate and simplify the difference quotient for the following functions:

(a.) $f(x) = 3x^2 + 2$,

$$\frac{f(a+h)-f(a)}{h} = \frac{3(a+h)^2+2-[3a^2+2]}{h}$$

$$= \frac{1}{h}\left(3(a^2+2ah+h^2)+2-(3a^2+2)\right)$$

$$= \frac{1}{h}\left(3a^2+6ah+3h^2+2-3a^2-2\right)$$

$$= \frac{1}{h}\left(6ah+3h^2\right)$$

$$= \boxed{6a+3h}$$

(b.) $f(x) = \frac{x}{x+1}$,

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{a+h}{a+h+1} - \frac{a}{a+1}}{h}$$

$$= \frac{1}{h}\left[\frac{(a+h)(a+1) - a(a+h+1)}{(a+h+1)(a+1)}\right]$$

$$= \frac{1}{h}\left[\frac{a^2+ah+a+h-a^2-ah-a}{(a+h+1)(a+1)}\right]$$

$$= \frac{1}{h}\left[\frac{h}{(a+h+1)(a+1)}\right]$$

$$= \boxed{\frac{1}{(a+h+1)(a+1)}}$$

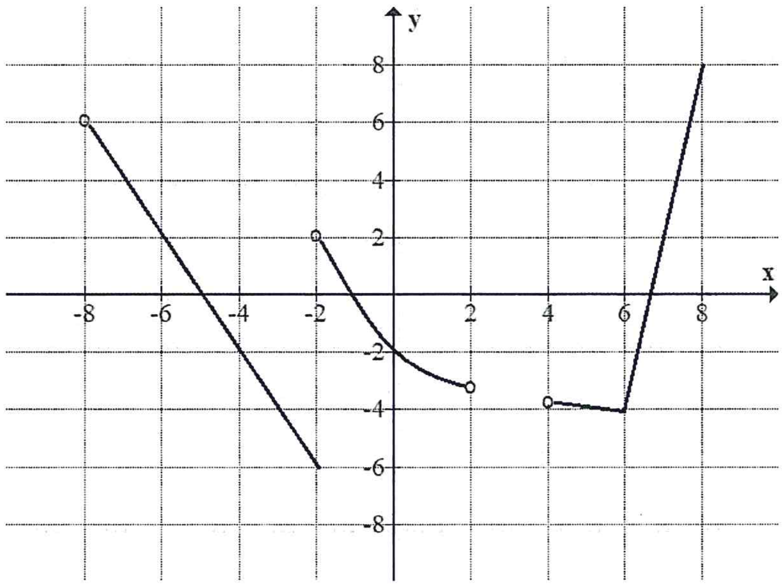
Problem 26: (4pts) Consider the graph $y = f(x)$ given below. Answer the following questions using interval notation (might need a union) where appropriate. Fill in the blanks:

(a.) the domain of $f(x) =$ $(-8, 2) \cup (4, 8]$.

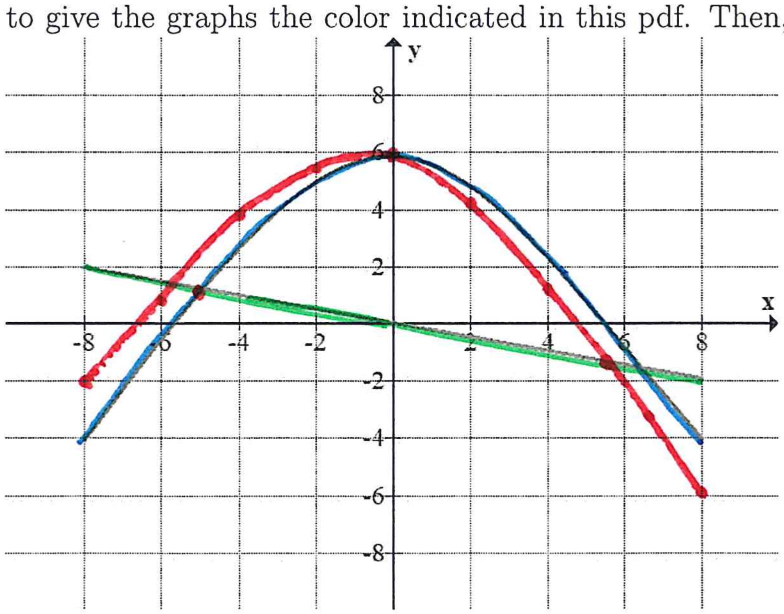
(b.) the range of $f(x) =$ $[-6, 8]$.

(c.) $f(0) =$ $\boxed{-2}$.

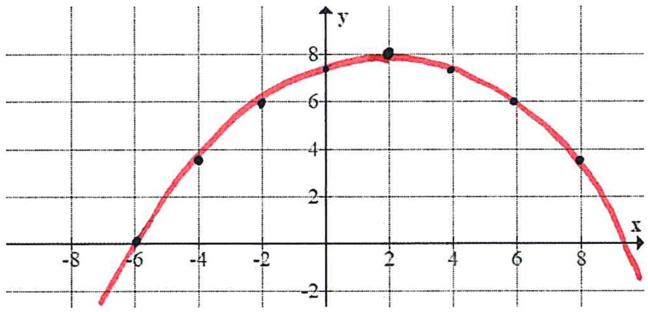
(d.) $f(3) =$ d.n.e..



Problem 27: (2pts) Suppose $y = f(x)$ is the blue graph given below and $y = g(x)$ is the green graph given below. Please use a colored pen, crayon, marker (whatever you have with color) to give the graphs the color indicated in this pdf. Then, graph $y = f(x) + g(x)$ in red.



Problem 28: (2pts) Let $f(x) = 8 - \frac{1}{8}(x-2)^2$ for $-6 \le x \le 6$.
Graph $y = f(x)$ and find the range of this function.



$f(0) = 8 - \frac{4}{8} = 7.5$

$f(-2) = 8 - \frac{16}{8} = 6$

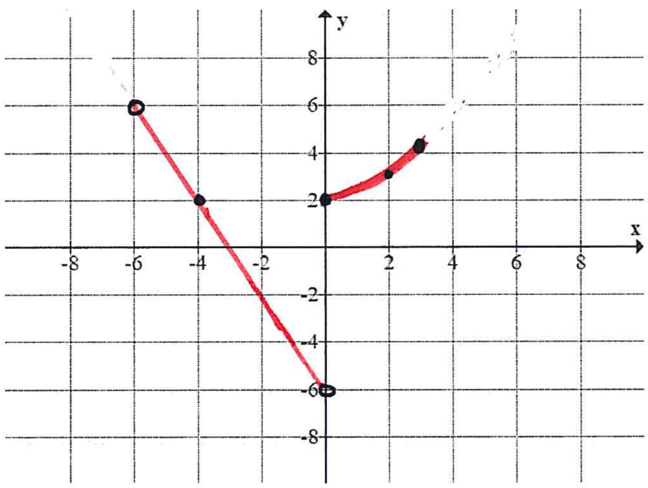
$f(4) = 8 - \frac{4}{8} = 7.5$

$f(-4) = 8 - \frac{36}{8} \approx 3.5$

$f(-6) = 8 - \frac{64}{8} = 0$

$\boxed{\text{range}(f) = (-\infty, 8]}$

Problem 29: (2pts) Let $f(x) = \begin{cases} -2x - 6 & : -6 < x < 0 \\ \frac{1}{4}x^2 + 2 & : 0 \le x \le 3 \end{cases}$.
Graph $y = f(x)$ in the plot below and find the domain and range of $f(x)$.



$\boxed{\text{dom}(f(x)) = (-6, 3]}$

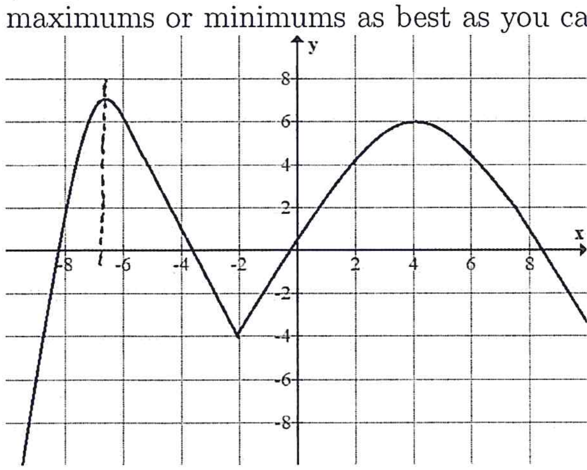
&

$\boxed{\text{range}(f(x)) = (-6, 6)}$

Problem 30: (1pt) For the previous problem, calculate $f(-2)$ and $f(2)$.

$f(-2) = -2(-2) - 6 = \boxed{-2}$ & $f(2) = \frac{1}{4}(2)^2 + 2 = \boxed{3}$

Problem 31: (3pts) Find the intervals of increase and the intervals of decrease. Also find any local maximums or minimums as best as you can given the plot below:

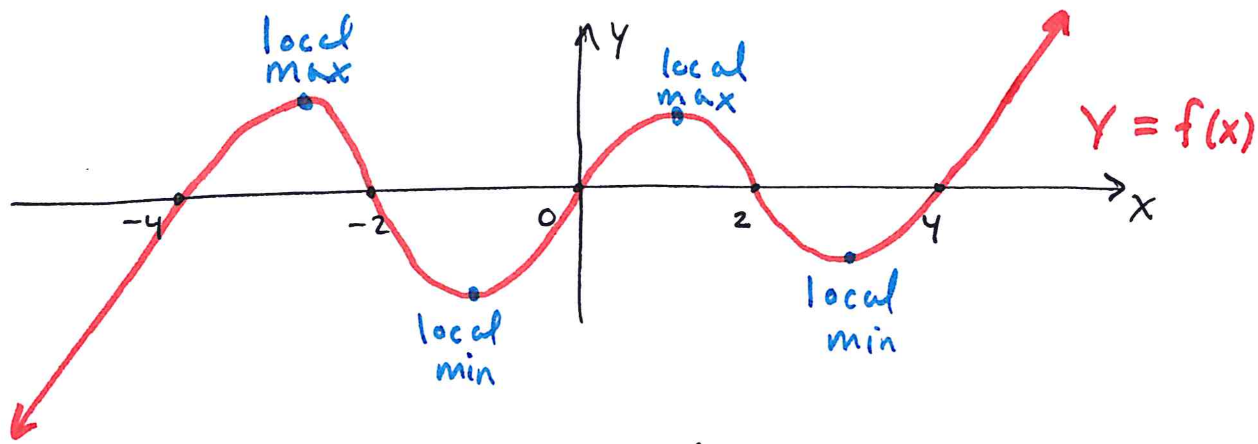


increase: $(-\infty, -6.8) \cup (-2, 4)$

decrease: $(-6.8, -2) \cup (4, \infty)$

local max of 7 at $x = -6.8$
local min of -4 at $x = -2$
local max of 6 at $x = 4$.

Problem 32: (2pts) Consider the function $f(x) = x(x+2)(x-2)(x+4)(x-4)$. Sketch the graph $y = f(x)$ and determine how many local minimums and maximums are found on the graph. Crosses zero (x-axis) at $x = 0, -2, 2, -4, 4$



- There are two local maximums.
- There are two local minimums.

Problem 33: (3pts) Consider $f(t) = t^2 + 3$ for $-5 \le t \le 5$. Find:

(a.) the average rate of change from $t = -3$ to $t = -2$ is $\boxed{-5}$.

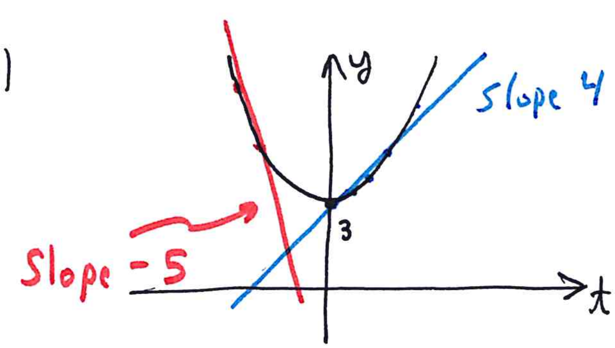
(b.) the average rate of change from $t = 1$ to $t = 3$ is $\boxed{4}$.

(c.) sketch $y = f(t)$ in the ty-plane and explain where the function is increasing and where it is decreasing. Do your answers to (a.) and (b.) make sense ?

(a.) $\frac{\Delta y}{\Delta t} = \frac{f(-2) - f(-3)}{-2-(-3)} = \frac{(-2)^2 + 3 - [(-3)^2 + 3]}{1} = 4 - 9 = \boxed{-5}$

(b.) $\frac{\Delta y}{\Delta t} = \frac{f(3) - f(1)}{3-1} = \frac{3^2 + 3 - (1^2 + 3)}{2} = \frac{8}{2} = \boxed{4}$

(c.)



answers above do make sense,

rate > 0 where increasing

rate < 0 where decreasing.

Problem 34: (6pts) Suppose $f(2) = 3$ and $g(2) = 7$ and $g(3) = 10$ and $f(7) = 0$. Calculate the following:

(a.) $(f+g)(2) =$ $f(2) + g(2) = 3+7 = \boxed{10}$

(b.) $(f-g)(2) =$ $f(2) - g(2) = 3-7 = \boxed{-4}$

(c.) $(fg)(2) =$ $f(2)\, g(2) = 3 \cdot 7 = \boxed{21}$

(d.) $\left(\frac{f}{g}\right)(2) =$ $f(2)/g(2) = \boxed{3/7}$.

(e.) $(f \circ g)(2) =$ $f(g(2)) = f(7) = \boxed{0}$

(f.) $(g \circ f)(2) =$ $g(f(2)) = g(3) = \boxed{10}$

Problem 35: (4pts) Let $f(x) = \sqrt[3]{x+6}$ and $g(x) = \sqrt{2x-9}$. Find the domain and formula for each of the following: notice f/g is another notation for $\frac{f}{g}$ just like $2/3$ can be written $\frac{2}{3}$.

(a.) $(f+g)(x) =$ $\sqrt[3]{x+6} + \sqrt{2x-9}$ and $\text{dom}(f+g) =$ $[9/2, \infty)$.

(b.) $(f/g)(x) =$ $\dfrac{\sqrt[3]{x+6}}{\sqrt{2x-9}}$ and $\text{dom}(f/g) =$ $(9/2, \infty)$.

$\text{dom}(f) = (-\infty, \infty)$ because cube roots allow negative inputs.

$2x - 9 \geq 0 \Rightarrow 2x \geq 9 \Rightarrow x \geq 9/2$, $\text{dom}(g) = [9/2, \infty)$

Problem 36: (2pts) Let $f(x) = \sqrt{-x}$ and let $g(x) = \sqrt{2+x}$. Find the domain and formula for $f+g$.

$-x \geq 0$ | $2 + x \geq 0$

$x \leq 0$ | $x \geq -2$

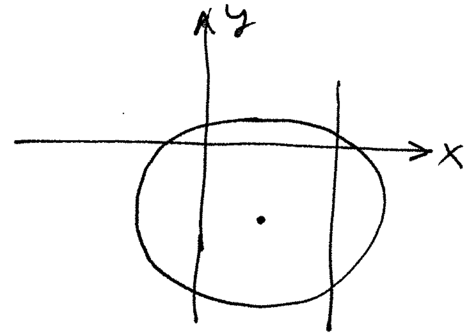
$\text{dom}(f) = (-\infty, 0]$ | $\text{dom}(g) = [-2, \infty)$

$$\boxed{\begin{array}{c} (f+g)(x) = \sqrt{-x} + \sqrt{2+x} \\ \text{dom}(f+g) = [-2, 0] \end{array}}$$

Problem 37: (1pts) Suppose a graph in the xy-plane is defined by $x^2 - 6x + y^2 + 8y = 0$. Can we view the graph of the equation as the graph of a function $y = f(x)$.

$$x^2 - 6x + y^2 + 8y = 0$$
$$(x-3)^2 + (y+4)^2 = 9 + 16 = 25$$

NOT GRAPH OF FUNCTION, FAILS VERTICAL LINE TEST.



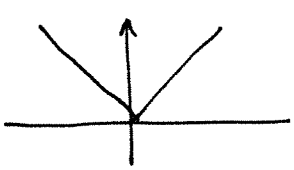
Problem 38: (1pts) Suppose a graph in the xy-plane is defined by $x^2y + 2 = y$. Can we view the graph of the equation as the graph of a function $y = f(x)$.

$$x^2y - y = -2$$
$$y(x^2 - 1) = -2$$
$$\underline{y = \frac{-2}{x^2-1} = f(x)}$$

Yes, it's a graph.

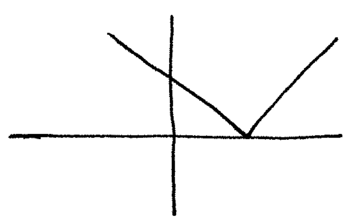
Problem 39: (3pts) Begin with $y = f(x)$ and provide a sequence of transformations which produces the graph $y = g(x)$ given that:

(a.) $f(x) = |x|$ and $g(x) = 3 - |x - 4|$



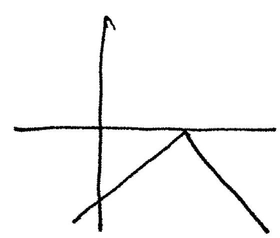
$y = |x|$

$\xrightarrow[4]{\text{RIGHT}}$



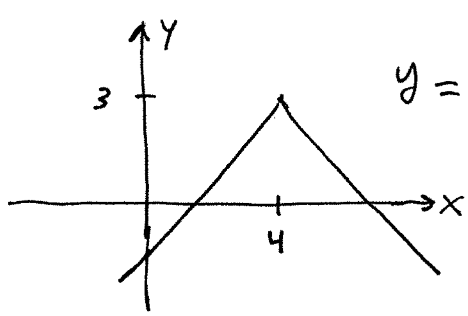
$y = |x-4|$

$\xrightarrow[\text{X-AXIS}]{\text{FLIP}}$ OVER



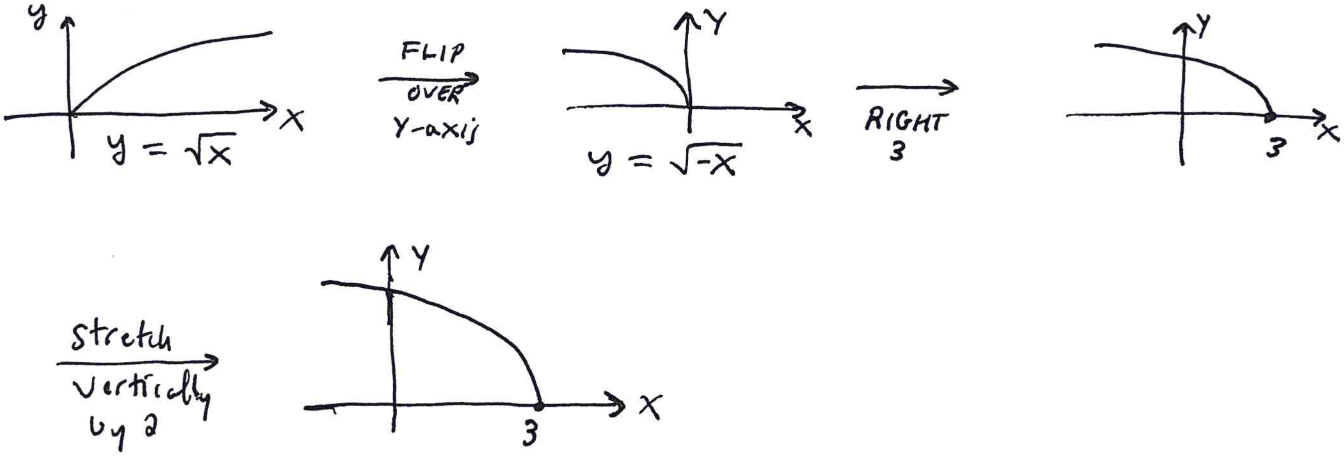
$y = -|x-4|$

$\xrightarrow[3]{\text{UP}}$



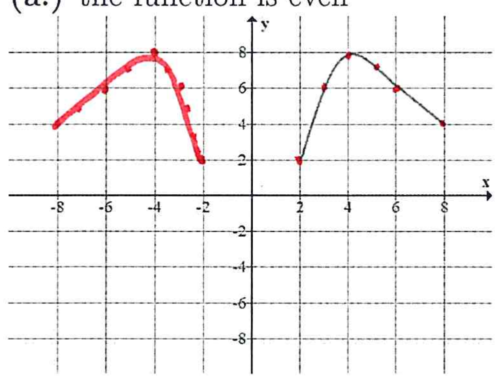
$y = 3 - |x-4|$

(b.) $f(x) = \sqrt{x}$ and $g(x) = 2\sqrt{3-x}$ $= 2\sqrt{-(x-3)}$

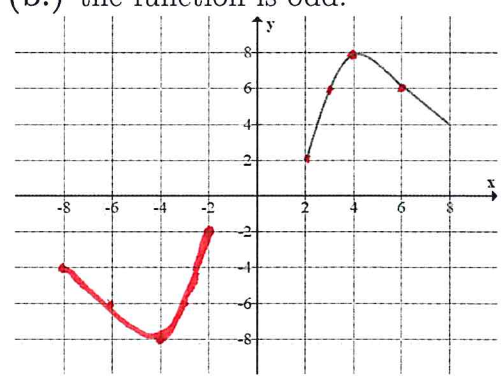


Problem 40: (2pts) Finish drawing the graphs under the assumption:

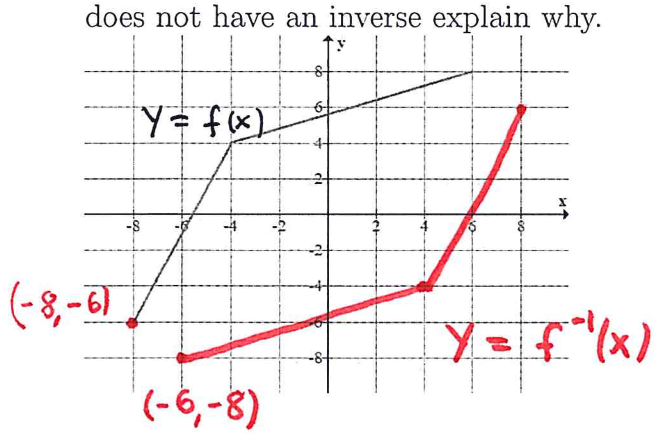
(a.) the function is even

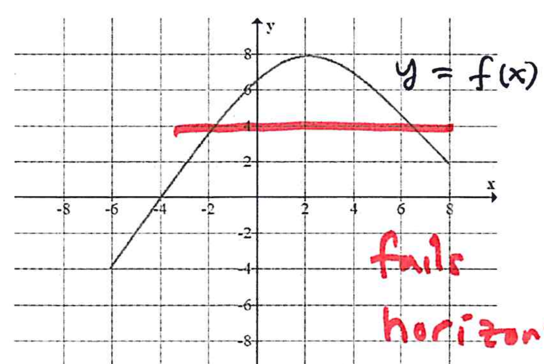


(b.) the function is odd.



Problem 41: (2pts) If possible, graph the inverse function for each function graph below. If the function does not have an inverse explain why.





fails horizontal line test.

NOT 1-1 $\therefore$ f^{-1} d.n.e.

Problem 42: (1pts) Let $f(x) = \dfrac{1}{3x+2}$. Show $f(x)$ is a one-to-one functionby showing that $f(a) = f(b)$ implies $a = b$.

$$f(a) = f(b) \Rightarrow \frac{1}{3a+2} = \frac{1}{3b+2}$$

$$\Rightarrow 3b+2 = 3a+2$$

$$\Rightarrow 3b = 3a$$

$$\Rightarrow b = a.$$

Problem 43: (4pts) For each formula given below identify an outside function $f(x)$ and an inside function $g(x)$ for which:

(a.) $(f \circ g)(x) = \sqrt{x^2+3x+2}$ has $f(x) =$ $\sqrt{x}$ and $g(x) =$ x^2+3x+2.

$\sqrt{x+2}$ $\quad$ x^2+3x

(b.) $(f \circ g)(x) = \dfrac{1}{3+\sqrt{x}}$ has $f(x) =$ $\dfrac{1}{x}$ and $g(x) =$ $3+\sqrt{x}$.

$\dfrac{1}{3+x}$ $\quad$ $\sqrt{x}$

(c.) $(f \circ g)(x) = 2 + (3+\sqrt{x})^3$ has $f(x) =$ $2+x^3$ and $g(x) =$ $3+\sqrt{x}$.

$2+(x+3)^3$ $\quad$ $\sqrt{x}$

(d.) $(f \circ g)(x) = \dfrac{1}{(3x-7)^2}$ has $f(x) =$ $\dfrac{1}{x^2}$ and $g(x) =$ $3x-7$.

$\dfrac{1}{(x-7)^2}$ $\quad$ $3x$

Remark: there are many other correct answers possible, I give 2nd answer in red

Problem 44: (1pts) Suppose f is an invertible function and $f(2) = 3$. Calculate $f^{-1}(3)$.

$$f(2) = 3 \Rightarrow f^{-1}(f(2)) = f^{-1}(3)$$

$$\therefore \boxed{f^{-1}(3) = 2}$$

Problem 45: (8pts) Given the function $f(x)$ calculate the formula for $f^{-1}(y)$.

(a.) $f(x) = 3x - 8$

(b.) $f(x) = 3 - \dfrac{1}{x-2}$

(c.) $f(x) = x^2 + 6$ given that $x \geq 0$

(d.) $f(x) = 6 + \dfrac{1}{\sqrt[3]{x-4}}$

(a.) $y = 3x - 8$

$3x = y + 8$

$x = \frac{1}{3}(y + 8)$

$\therefore \boxed{f^{-1}(y) = \frac{1}{3}(y + 8)}$

(b.) $y = 3 - \dfrac{1}{x-2}$

$y - 3 = \dfrac{-1}{x-2}$

$x - 2 = \dfrac{-1}{y-3} = \dfrac{1}{3-y}$

$x = 2 + \dfrac{1}{3-y}$

$\therefore \boxed{f^{-1}(y) = 2 + \dfrac{1}{3-y}}$

(c.) $y = x^2 + 6$, $x \geq 0$

$x^2 = y - 6$

$x = \pm\sqrt{y-6}$

but, $x \geq 0$ $\therefore$ $x = \sqrt{y-6}$

Hence, $\boxed{f^{-1}(y) = \sqrt{y-6}}$

(d.) $y = 6 + \dfrac{1}{\sqrt[3]{x-4}}$

$y - 6 = \dfrac{1}{\sqrt[3]{x-4}}$

$\left(\sqrt[3]{x-4}\right)^3 = \left(\dfrac{1}{y-6}\right)^3$

$x - 4 = \dfrac{1}{(y-6)^3}$

$x = 4 + \dfrac{1}{(y-6)^3}$

$\boxed{f^{-1}(y) = 4 + \dfrac{1}{(y-6)^3}}$