NAME ____

MATH 113-03: FALL 2020	QUIZ 1

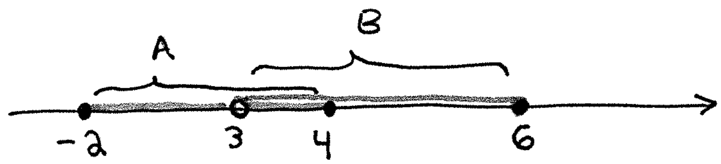
You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. Thanks!

Problem 1: (1pt) Multiply the following expressions and collect like power terms to give your answer as a polynomial in standard form:

$$
\begin{aligned}
(x-3)^2(x^2+1) &= (x^2-6x+9)(x^2+1) \\
&= x^4-6x^3+9x^2+x^2-6x+9 \\
&= \boxed{x^4-6x^3+10x^2-6x+9}
\end{aligned}
$$

Problem 2: (3pt) Let $A = \{x \in \mathbb{R} \mid |x-1| \leq 3\}$ and $B = (3, 6]$. Picture A and B on a number line and find $A \cup B$ and $A \cap B$ in interval notation.

Notice $|x-1| \leq 3 \iff -3 \leq x-1 \leq 3 \iff \underbrace{-2 \leq x \leq 4}_{\text{describes } x \in A}$



$$
\boxed{\begin{aligned} A \cup B &= [-2, 6] \\ A \cap B &= (3, 4] \end{aligned}}
$$

Problem 3: (2pt) Find the domain of the expression $\sqrt{3x-21}$. Provide answer in interval notation.

Need $3x - 21 \geq 0$

$3x \geq 21$

$x \geq 21/3 = 7 \quad \therefore \quad x \geq 7 \Rightarrow \boxed{[7, \infty)}$

Problem 4: (2pt) Assume $x, y > 0$ and use laws of algebra to determine A, B as indicated below:

$$x^A y^B = \sqrt[3]{\frac{y^{-2}(xy)^2}{x^{-1}y^3}}$$

$$= \left(\frac{y^{-2}x^2y^2}{x^{-1}y^3}\right)^{1/3}$$

$$= (x^3y^{-3})^{1/3}$$

$$= x^{3/3}y^{-3/3}$$

$$= x^1y^{-1} \quad \therefore \quad \boxed{A = 1} \ \& \ \boxed{B = -1}$$

Problem 5: (2pt) Peform the addition and simplify the resulting expression.

$$\frac{3-x}{2x+1} + 3 = \frac{3 - x + 3(2x+1)}{2x+1}$$

$$= \frac{3 - x + 6x + 3}{2x+1}$$

$$= \boxed{\frac{6 + 5x}{2x+1}}$$

Remark: I clarified the missing $f(x)$ in class,

Problem 6: (4pt) **Factor** each $f(x)$ given below completely over $\mathbb{R}$ and **solve** $f(x) = 0$.

(a.) $\underbrace{2x^4 + x^3}_{f(x)}, = \boxed{x^3(2x+1)} = 2x^3\left(x+\frac{1}{2}\right)$ also helpful

$$f(x) = x^3(2x+1) = 0 \quad \begin{cases} x^3 = 0 \Rightarrow \boxed{x=0} \\ 2x+1=0 \Rightarrow \boxed{x = -1/2} \end{cases}$$

(b.) $\underbrace{x^4 + 2x^2 - 24}_{f(x)}. = (x^2+6)(x^2-4)$

$= \boxed{(x^2+6)(x+2)(x-2)}$ ← factored completely over $\mathbb{R}$ since x^2+6 cannot be further reduced.. unless we use $\mathbb{C}$ $x^2+6 = (x-i\sqrt{6})(x+i\sqrt{6})$

$f(x) = 0 \Rightarrow \boxed{x = \pm 2}$

$\underbrace{x^2+6=0}_{x^2=-6}$ no real sol$^{\text{n}}$ $\quad \underbrace{x+2=0}_{x=-2} \quad \underbrace{x-2=0}_{x=2}$

Problem 7: (4pt) For each quadratic polynomial $f(x)$ given below, complete the square and find all real or complex solutions of $f(x) = 0$:

(a.) $f(x) = x^2 + 8x + 6,$

$= (x+4)^2 - 16 + 6$

$= \boxed{(x+4)^2 - 10}$

$$f(x) = (x+4-\sqrt{10})(x+4+\sqrt{10}) = 0 \quad \begin{cases} x = -4+\sqrt{10} \\ x = -4-\sqrt{10} \end{cases}$$

(b.) $f(x) = x^2 + 10x + 27.$

$= (x+5)^2 - 25 + 27$

$= \boxed{(x+5)^2 + 2}$ ← completed square

$= (x+5)^2 - (i\sqrt{2})^2$

Thus, $f(x) = (x+5-i\sqrt{2})(x+5+i\sqrt{2}) = 0$

$\Rightarrow \boxed{x = -5 \pm i\sqrt{2}}$

Problem 8: (3pt) **Choose one** of the the following equations to solve over $\mathbb{R}$,

(a.) $|2x+1| = 11$

(b.) $\sqrt{x+1} - \sqrt{x-4} = 1$

(a.)

$$|2x+1| = 11$$
$$2x+1 = \pm 11$$
$$2x = \pm 11 - 1$$
$$x = \frac{-1 \pm 11}{2}$$
$$x = \frac{-1+11}{2} \quad \text{or} \quad \frac{-1-11}{2}$$
$$\boxed{x = 5} \quad \text{or} \quad \boxed{x = -6}$$

(b.)

$$\sqrt{x+1} - \sqrt{x-4} = 1$$
$$\left(\sqrt{x+1}\right)^2 = \left(1 + \sqrt{x-4}\right)^2$$
$$x+1 = 1 + 2\sqrt{x-4} + x - 4$$
$$4 = 2\sqrt{x-4}$$
$$(2)^2 = \left(\sqrt{x-4}\right)^2$$
$$4 = x - 4$$
$$\therefore \boxed{x = 8}$$

aww, I wanted there to be extraneous sol^{n}'s. Still we should check,

$$\sqrt{8+1} - \sqrt{8-4} = \sqrt{9} - \sqrt{4} = 3 - 2 = 1 \checkmark$$

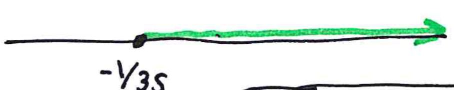
Problem 9: (3pts) **Solve one** of the inequalities below and express your answer in interval notation as well as with a number line picture.

(a.) $\frac{1}{4} - \frac{1}{12}x \geq \frac{1}{6} - 3x,$

(b.) $|2x-1| > 5,$

(a.) Multiply by 12,

$$12\left(\frac{1}{4} - \frac{1}{12}x\right) \geq 12\left(\frac{1}{6} - 3x\right)$$
$$\frac{12}{4} - x \geq 12\left(\frac{1}{6}\right) - 12(3x)$$
$$3 - x \geq 2 - 36x$$
$$35x \geq -1$$
$$x \geq -1/35$$



$$\boxed{[-1/35, \infty)}$$

(b.) $|2x-1| > 5$

$$2x-1 < -5 \quad \text{or} \quad 2x-1 > 5$$
$$2x < -4 \quad \text{or} \quad 2x > 6$$
$$x < -2 \quad \text{or} \quad x > 3$$

$$\boxed{(-\infty, -2) \cup (3, \infty)}$$

Problem 10: (3pts) Solve the following inequality using an appropriate technique. Show your work and write the answer using interval notation (you might need to use ∪ for union)

$$\underbrace{\frac{(x+4)^2(x+2)^3}{x^2(x-3)^5}}\geq 0$$

THANKS TO ME FOR ALREADY HAVING IT FACTORED ☺

Algebraic critical #'s $-4, -2, 0, 3$

power of $(x-a)$: even odd even odd

$+ + + +$ | $+ + + +$ | $- - - - -$ | $- - - - -$ | $+ + +$

-4 $\quad -2$ $\quad 0$ $\quad 3$

$(-\infty, -4] \cup [-4, -2] \cup (3, \infty)$ ← need to exclude since division by zero = ☹

$$\boxed{(-\infty, -2] \cup (3, \infty)}$$

Problem 11: (2pts) Use completing the square and algebra as needed to place each circle equation below into standard form. Find the center and radius of the circle.

$$x^2 - 4x + y^2 + 12y = 12$$

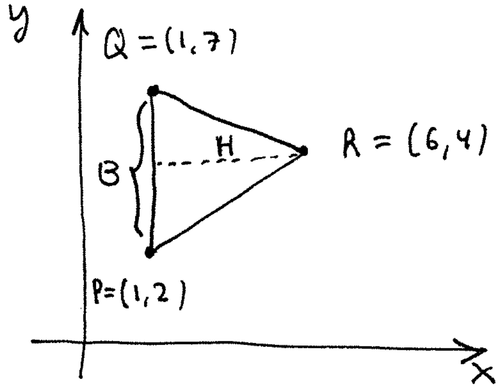
$(x-2)^2 - 4 + (y+6)^2 - 36 = 12$

$(x-2)^2 + (y+6)^2 = 12 + 4 + 36 = 52.$

$\boxed{(x-2)^2 + (y+6)^2 = (\sqrt{52})^2}$ ← standard form

$\boxed{\begin{array}{l}\text{Center at } (2, -6) \\ \text{radius } R = \sqrt{52}\end{array}}$

Problem 12: (1pt) Find the area of the triangle with vertices $P = (1, 2)$, $Q = (1, 7)$ and $R = (6, 4)$. Show your work including appropriate diagrams.



$$\text{BASE} = B = 7 - 2 = 5$$

$$H = 6 - 1 = 5$$

$$\text{AREA} = \frac{1}{2} BH = \boxed{\frac{25}{2}}$$

Problem 13: (1pt) Consider the equation $\frac{x^2-1}{xy} = 0$. Is $P = (0, 0)$ on the graph of the equation ? Is $Q = (1, 3)$on the graph of the given equation ?

- No, $P = (0,0)$ is not in domain of $\frac{x^2-1}{xy}$ so it cannot solve $\frac{x^2-1}{xy} = 0$.
- YES, $Q = (1,3)$ is on the graph of $\frac{x^2-1}{xy} = 0$ since $\frac{1^2-1}{3} = \frac{0}{3} = 0$. ✓

Problem 14: (1pts) Find the equation of a line given that contains points $(2, -1)$ and $(3, 5)$.

$$y = -1 + m(x-2) \quad \text{where} \quad m = \frac{\Delta y}{\Delta x} = \frac{5-(-1)}{3-2} = \frac{6}{1} = 6$$

$$\therefore \boxed{y = -1 + 6(x-2)}$$

aka

$$\boxed{y = 6x - 13}$$