NAME ______________________________

MATH 113-08: FALL 2020 — QUIZ 1

You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. Please BOX *your answer to each question. At least 25pts to earn here. Thanks!*

Problem 1: (2pts) Find the equation of a line given that contains points $\underbrace{(2,-1)}_{P_1}$ and $\underbrace{(3,5)}_{P_2}$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-1)}{3-2} = \frac{6}{1} = 6$$

$$\text{Thus } \underbrace{y = y_1 + m(x - x_1)}_{\text{point-slope formula for line.}} = -1 + 6(x-2) = -1 + 6x - 12$$

$$\therefore \boxed{y = 6x - 13}$$

Problem 2: (2pt) Let $A = \{x \in \mathbb{R} \mid |x-1| > 3\}$. Write A in interval notation (use union if needed).

$$|x-1| > 3 \iff x-1 < -3 \text{ OR } x-1 > 3$$
$$\iff x < -2 \text{ OR } x > 4$$
$$\iff x \in (-\infty, -2) \text{ or } x \in (4, \infty)$$
$$\iff x \in (-\infty, -2) \cup (4, \infty)$$
$$\therefore \boxed{A = (-\infty, -2) \cup (4, \infty)}$$

Problem 3: (2pt) Find the domain of the expression $\dfrac{1}{\sqrt{10-2x}}$. Provide answer in interval notation.

We need $10 - 2x > 0 \Rightarrow 10 > 2x \Rightarrow 5 > x \Rightarrow x < 5$

Therefore the domain of the expression is $\boxed{(-\infty, 5)}$

Problem 4: (2pt) Assume $x, y > 0$ and use laws of algebra to determine A, B as indicated below:

$$x^A y^B = \left(\frac{y^{-2}}{x^{-1}y^3}\sqrt[3]{xy}\right)^3$$

$$= \left(x\, y^{-2-3}\, \sqrt[3]{xy}\right)^3$$

$$= \left(x\, y^{-5} \sqrt[3]{xy}\right)^3$$

$$= x^3 \left(y^{-5}\right)^3 \left(\sqrt[3]{xy}\right)^3$$

$$= x^3\, y^{-15}\, x\, y$$

$$= \underline{x^4\, y^{-14}} \quad \therefore \quad \boxed{A = 4,\ B = -14}$$

Problem 5: (2pt) Peform the addition and simplify the resulting expression.

$$\frac{3-x}{7-x} + 2 = \frac{3-x}{7-x} + 2\left(\frac{7-x}{7-x}\right)$$

$$= \frac{3-x}{7-x} + \frac{14-2x}{7-x}$$

$$= \frac{3-x+14-2x}{7-x}$$

$$= \boxed{\frac{17-3x}{7-x}}$$

Problem 6: (4pt) **Factor** each $f(x)$ given below completely over $\mathbb{R}$ and **solve** $f(x) = 0$.

(a.) $\underbrace{x^2 - 9x + 20}$,

$$20 = 10 \cdot 1 = 4 \cdot 5 = (-4)(-5)$$

$$f(x) = x^2 - 9x + 20 = \boxed{(x-4)(x-5)}$$ factored completely.

$$f(x) = 0 \rightarrow (x-4)(x-5) = 0 \Rightarrow \boxed{x = 4, 5}$$

(b.) $\underbrace{(x^2 + 6x + 9)}_{(x-3)^2}\underbrace{(x^2 + 4x + 5)}$.

$(x-3)^2$ complete the square to $(x+2)^2 + 1$.

$$f(x) = \boxed{(x-3)^2((x+2)^2+1)} = \boxed{(x-3)^2(x^2+4x+5)}$$

either fine, these are f(x) complet factored.

$$f(x) = 0 \Rightarrow \boxed{x = 3}$$

Problem 7: (6pt) For each quadratic polynomial $f(x)$ given below, complete the square and find all real or complex solutions of $f(x) = 0$:

(a.) $f(x) = 2x^2 - 20x + 50$,

$$= 2(x^2 - 10x + 25)$$

$$= \underline{2(x-5)^2}.$$

Then $f(x) = 0 = 2(x-5)^2$

$$\Rightarrow \boxed{x = 5}$$

(b.) $f(x) = x^2 - 12x + 39$.

$$= (x-6)^2 - 36 + 39$$

$$= \underline{(x-6)^2 + 3}$$ Square complete.

$$f(x) = 0 \longrightarrow (x-6)^2 = -3$$

$$\therefore \quad x - 6 = \pm\sqrt{-3} = \pm i\sqrt{3}$$

$$\boxed{x = 6 \pm i\sqrt{3}}$$

Problem 8: (4pt) Choose one of the the following equations to solve over $\mathbb{R}$,

(a.) $|2x+1| = 11$

(b.) $\sqrt{x+1} - \sqrt{x-4} = 1$

$$|2x+1| = 11$$
$$2x+1 = \pm 11$$
$$2x = \pm 11 - 1$$
$$x = \frac{\pm 11 - 1}{2} = \frac{-11-1}{2} \text{ or } \frac{11-1}{2}$$
$$\boxed{x = -6 \text{ or } 5}$$

$$\sqrt{x+1} - \sqrt{x-4} = 1$$
$$\left(\sqrt{x+1}\right)^2 \overset{2}{=} \left(1 + \sqrt{x-4}\right)^2$$
$$x+1 = 1 + 2\sqrt{x-4} + \left(\sqrt{x-4}\right)^2$$
$$x+1 = 1 + 2\sqrt{x-4} + x - 4$$
$$4 = 2\sqrt{x-4}$$
$$(2)^2 \overset{2}{=} \left(\sqrt{x-4}\right)^2$$
$$4 = x - 4 \quad \therefore \boxed{x = 8}$$

Check: $\sqrt{8+1} - \sqrt{8-4} = \sqrt{9} - \sqrt{4}$
$= 3 - 2$
$\neq 1$

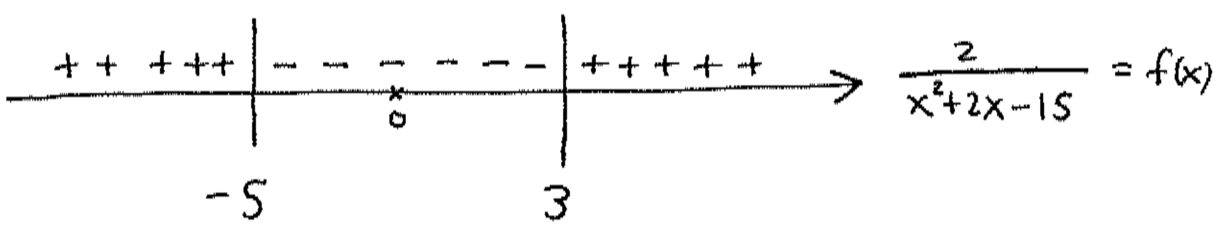
(it is important to check, there are many cases where no sol^{n}s work)

Problem 9: (4pts) Solve the following inequality using an appropriate technique. Show your work and write the answer using interval notation (you might need to use $\cup$ for union)

$$\frac{2}{x^2+2x-15} > 0$$

observe $x^2 + 2x - 15 = (x+5)(x-3)$

So Algebraic Critical #'s are $-5, 3$



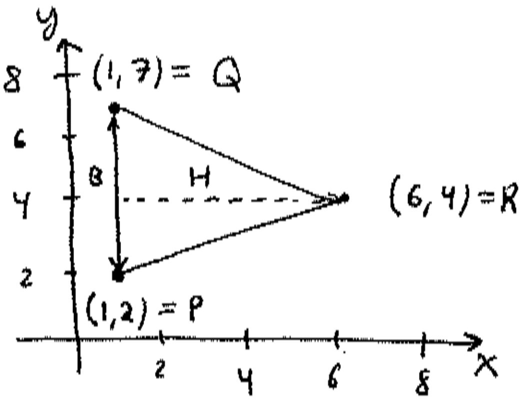
You can check $f(0) = \frac{2}{0+0-15} = \frac{2}{-15} < 0$

Then since $(x+5)$ & $(x-3)$ are odd-power factors get a sign-flip

Anyway, the answer is: (Do NOT INCLUDE $x = -5, 3$, don't want division by ZERO!)

$$\boxed{(-\infty, -5) \cup (3, \infty)}$$

Problem 10: (2pt) Find the area of the triangle with vertices $P = (1, 2)$, $Q = (1, 7)$ and $R = (6, 4)$. Show your work including appropriate diagrams.



$$B = 7 - 2 = 5$$
$$H = 6 - 1 = 5$$
$$\text{AREA} = \tfrac{1}{2} BH = \boxed{\frac{25}{2}}$$

Problem 11: (2pts) Use completing the square and algebra as needed to place each circle equation below into standard form. Find the center and radius of the circle.

$$x^2 - 4x + y^2 + 12y = 2$$

$$(x-2)^2 - 4 + (y+6)^2 - 36 = 2$$

$$(x-2)^2 + (y+6)^2 = 2 + 4 + 36 = 42 = (\sqrt{42})^2$$

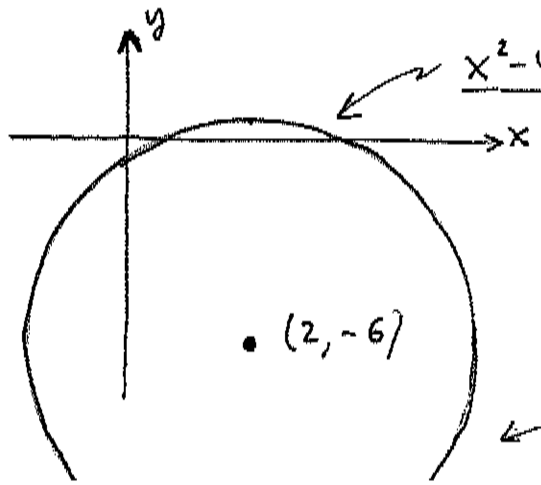
Compare against

$$(x-h)^2 + (y-k)^2 = R^2$$

To read off $h = 2$, $k = -6$ and $R = \sqrt{42}$.

Circle centered at $\boxed{(2,-6) \text{ with } R = \sqrt{42}}$

$x^2 - 4x + y^2 + 12y = 2$.



$\sqrt{42} \approx 6.48$

poor planning on my part ☺.