You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. Thanks!

Problem 1: (4pts) Suppose $f(2) = 12$ and $g(2) = 30$, $f(3) = 6$ and $g(3) = 7$, $f(4) = 21$ and $g(4) = 0.5$. In addition, suppose $g(12) = 42$. Calculate the following:

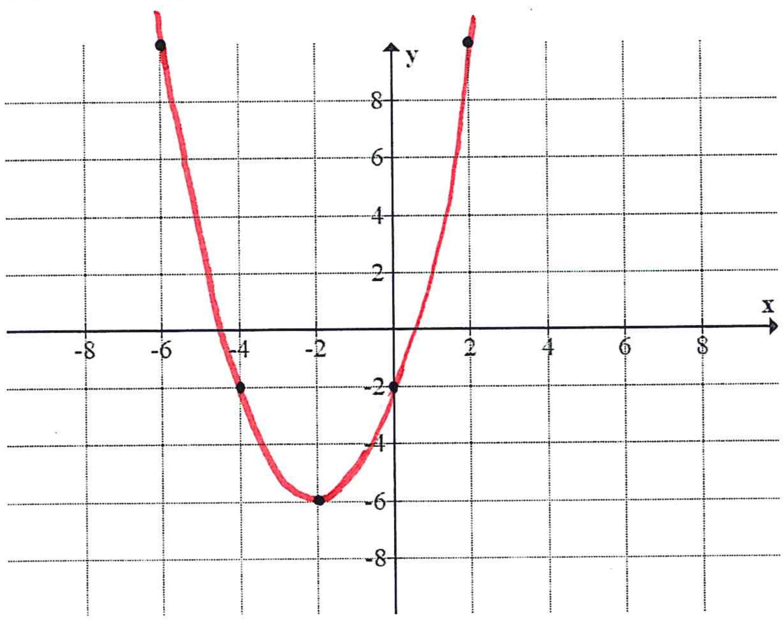
(a.) $(f+g)(2) =$ $f(2)+g(2) = \boxed{42}$.

(b.) $(fg)(3) =$ $f(3)g(3) = 6 \cdot 7 = \boxed{42}$

(c.) $\left(\frac{f}{g}\right)(4) =$ $f(4)/g(4) = \frac{21}{0.5} = \boxed{42}$

(d.) $(g \circ f)(2) =$ $g(f(2)) = g(12) = \boxed{42}$.

Problem 2: (2pts) Let $f(x) = x^2 + 4x - 2$. Carefully graph $y = f(x)$ on the grid provided below.



$y = f(x) = x^2 + 4x - 2$

$= (x+2)^2 - 4 - 2$

$= \underbrace{-6 + (x+2)^2}$

parabola centered at $(-2,-6)$ opens up.

$f(2) = -6 + 4^2 = 10.$

Problem 3: (1pts) Express the range of the function in the previous problem in interval notation.

$\boxed{\text{range}(f) = [-6, \infty)}$

Problem 4: (3pts) The difference quotient based at a for $f(x)$ is given by $\frac{f(a)-f(a+h)}{h}$ where $h \neq 0$. Calculate and simplify the difference quotient for $f(x) = 2x^2 - 7$.

$$\frac{f(a+h) - f(a)}{h} = \frac{2(a+h)^2 - 7 - [2a^2 - 7]}{h}$$

$$= \frac{2(a^2 + 2ah + h^2) - 7 - 2a^2 + 7}{h}$$

$$= \frac{2a^2 + 4ah + 2h^2 - 7 - 2a^2 + 7}{h}$$

$$= \frac{4ah + 2h^2}{h}$$

$$= \boxed{4a + 2h.}$$

Problem 5: (2pts) Let $f(x) = \begin{cases} x^2 + 3 & : -6 < x < 0 \\ 10 + \sqrt{x} & : 0 \leq x \leq 8 \end{cases}$.

$f(-2) = (-2)^2 + 3 = 7$

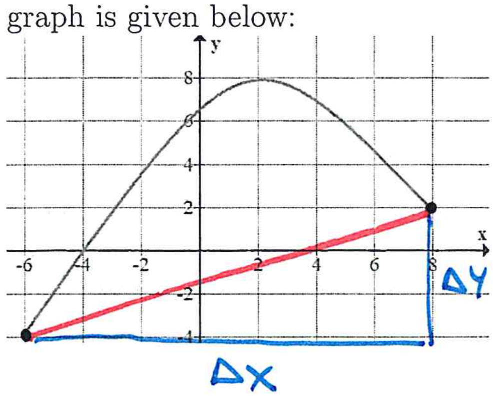
$f(4) = 10 + \sqrt{4} = 12$

Given the function above, calculate:

(a.) $f(-2) =$ $\boxed{7}$ ________.

(b.) $f(4) =$ $\boxed{12}$ ________.

Problem 6: (1pts) Find the average rate of change from $x = -6$ to $x = 8$ for the function whose graph is given below:



$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(-6)}{8 - (-6)}$$

$$= \frac{2 - (-4)}{14}$$

$$= \boxed{\frac{6}{14} = \frac{3}{7}}$$

Problem 7: (2pts) Given $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{4+2x}$, calculate the formula for $(f+g)(x)$ and find the domain of $f+g$.

dom $(f(x))$: $4 - x \geq 0$
$4 \geq x$
$x \leq 4$

dom $(g(x))$: $4 + 2x \geq 0$
$4 \geq -2x$
$-2 \leq x$

dom $(f) = (-\infty, 4]$ and dom $(g) = [-2, \infty)$

Then dom $(f+g)$ = dom $(f) \cap$ dom (g) = $\boxed{[-2, 4]}$

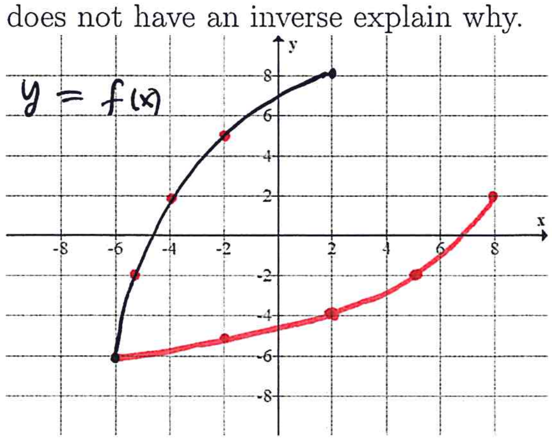
$$(f+g)(x) = f(x) + g(x) = \boxed{\sqrt{4-x} + \sqrt{4+2x}}$$

Problem 8: (2pts) For the functions given above, find the formula and domain for f/g.

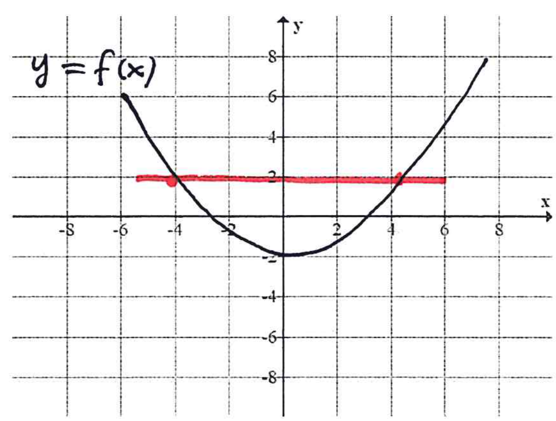
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{\sqrt{4-x}}{\sqrt{4+2x}}}$$

$\boxed{\text{dom}\,(f/g) = (-2, 4]}$ since $x = -2$ must be omitted due to division by zero.

Problem 9: (4pts) If possible, graph the inverse function for each function graph below. If the function does not have an inverse explain why.



$y = f^{-1}(x)$



NOT INVERTIBLE, FAILS HORIZONTAL LINE TEST, NOT 1-1.

Problem 10: (3pts) Let $f(x) = x^2 + 2$ and $g(x) = \dfrac{1}{x} + \sqrt{x}$. Find the formulas for:

(a.) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x} + \sqrt{x}\right) = \boxed{\left(\frac{1}{x} + \sqrt{x}\right)^2 + 2}$

(b.) $(g \circ f)(x) = g(f(x)) = g(x^2+2) = \boxed{\frac{1}{x^2+2} + \sqrt{x^2+2}}$

(c.) $(f \circ f)(x) = f(f(x)) = f(x^2+2) = \boxed{(x^2+2)^2 + 2}$

Problem 11: (2pts) For each formula given below fill in the blank as appropriate:

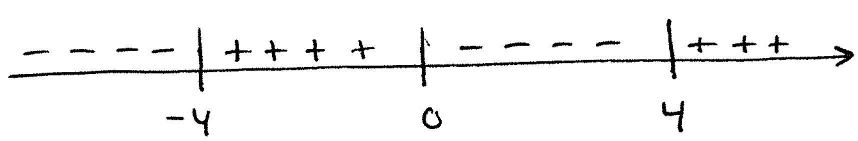
(a.) $(f \circ g)(x) = \sqrt{x^2 + 3x + 2}$ has $f(x) = \sqrt{x}$ and $g(x) = \boxed{x^2 + 3x + 2}$.

(b.) $(f \circ g)(x) = (x^2 + 3x - 9)^4$ has $f(x) = \boxed{x^4}$ and $g(x) = x^2 + 3x - 9$.

Problem 12: (4pts) Consider $f(x) = \sqrt{x^3 - 16x}$. Find the domain of the function in interval notation.

Need $x^3 - 16x \geq 0$. We solve this via # line technique,

$x(x^2 - 16) = x(x+4)(x-4) \geq 0$



$\therefore \boxed{\text{dom}(f) = [-4, 0] \cup [4, \infty)}$