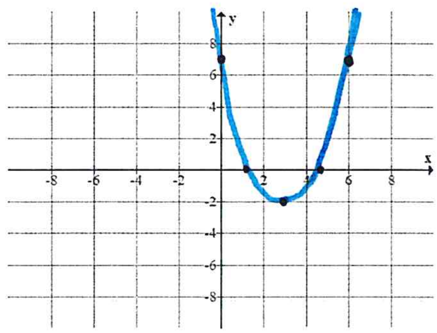
NAME: ______________________________.

MATH 113:	QUIZ 3

No phones. You are allowed a calculator and a sheet of notes front and back. 45 minutes to take this Quiz. At least 25pts to earn here. Thanks!

Problem 1: (4pts) Calculate the discriminant for each $f(x)$ given below and factor $f(x)$ over $\mathbb{R}$ if possible. In addition, graph $y = f(x)$ carefully in the plot provided:

(a.) $f(x) = x^2 - 6x + 7$ $= (x-3)^2 - 2 = (x-3-\sqrt{2})(x-3+\sqrt{2})$



vertex at $(3, -2)$

x-intercepts at $3 \pm \sqrt{2} \cong 4.4 \ \& \ 1.6$

y-intercept at 7

$\underbrace{b^2 - 4ac = 36 - 28 = 8 > 0}_{\text{makes sense, we factored it.}}$

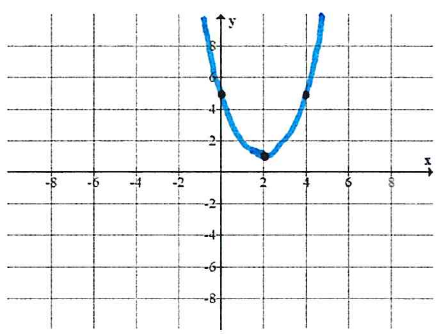
(b.) $f(x) = x^2 - 4x + 5$ $= (x-2)^2 + 1$, $b^2 - 4ac = 16 - 20 = -4 < 0$

$\Rightarrow$ cannot factor over $\mathbb{R}$

vertex at $(2, 1)$

y-intercept at $(0, 5)$

$f(6) = 4^2 + 1 = 17$ (off graph)



Problem 2: (2pts) Suppose a polynomial $P(x)$ has a graph which crosses the x-axis at $x = 3$ and bounces off the x-axis at $x = -2$. Find formula of $P(x)$ given that the y-intercept is 10.

$P(x) = A(x-3)(x+2)^2 \ \ \& \ \ P(0) = 10$

$= A(x-3)(x^2 + 4x + 4)$

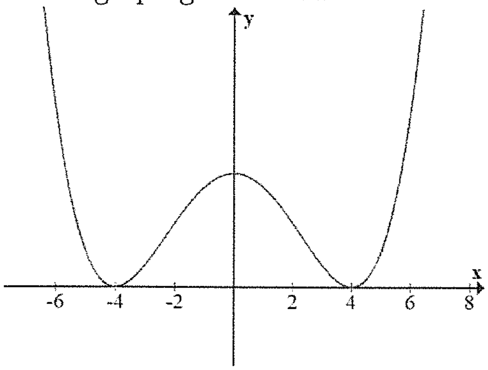
$= A(x^3 + 4x^2 + 4x - 3x^2 - 12x - 12)$

$= A(x^3 + x^2 - 8x - 12) \Rightarrow P(0) = -12A = 10$

$\therefore A = \frac{-10}{12} = \frac{-5}{6}$

$\boxed{P(x) = -\frac{5}{6}(x^3 + x^2 - 8x - 12)}$

Problem 3: (2pts) Find $P(x)$ which could have a graph which shares the same shape and x-intercepts as the graph given below:



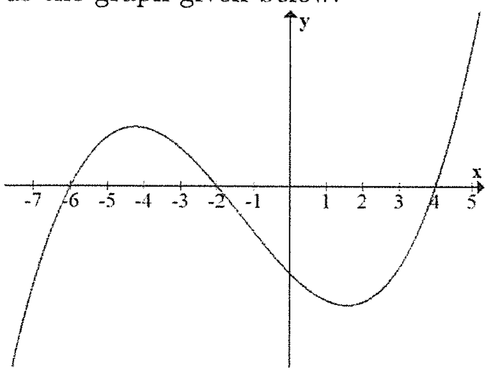
bounce at $x=-4 \Rightarrow (x+4)^2$ factor

bounce at $x=4 \Rightarrow (x-4)^2$ factor

$$\boxed{P(x) = (x+4)^2(x-4)^2}$$

(note $P(x) \cong x^4$ for $|x| >> 0$ matches the given graph)

Problem 4: (2pts) Find $P(x)$ which could have a graph which shares the same shape and x-intercepts as the graph given below:



$$\boxed{P(x) = (x+6)(x+2)(x-4)}$$

- $P(x) \approx x^3$ for $|x| \gg 0$ and this matches global picture for given graph.

Problem 5: (2pts) Let $P(x) = x^3 + x^2 - 4x - 4$. Show that -1 is a zero of $P(x)$ and find all the other zeros of $P(x)$. *Hint: factoring by grouping is a good idea here*

$$P(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 = -1 + 1 + 4 - 4 = 0.$$

$$P(x) = x^2(x+1) - 4(x+1) = (x^2-4)(x+1)$$

thus, $P(x) = (x-2)(x+2)(x+1) \longrightarrow$ $\boxed{\text{Zeros of } P(x) \text{ include } 2, -2 \text{ and } -1}$

Problem 6: (3pts) Factor $f(x) = x^4 + 7x^3 + 7x^2 + 7x + 6$ completely over $\mathbb{R}$. Hint: $f(i) = 0$.

$f(i) = 0 \Rightarrow x^2+1$ factors $f(x)$.

$$
\begin{array}{r|l}
 & x^2 + 7x + 6 \\
\hline
x^2+1 & x^4 + 7x^3 + 7x^2 + 7x + 6 \\
 & -(x^4 \qquad\quad + x^2) \\
\hline
 & \quad 7x^3 + 6x^2 + 7x + 6 \\
 & \quad -(7x^3 \qquad + 7x) \\
\hline
 & \qquad\quad 6x^2 + 6 \\
 & \qquad\quad 6x^2 + 6 \\
\hline
 & \qquad\qquad\quad \boxed{0}
\end{array}
$$

$$f(x) = (x^2+1)(x^2+7x+6)$$
$$= \boxed{(x^2+1)(x+1)(x+6)}$$

Problem 7: (1pts) If $R(x) = 3x^5 + 3x^3 + 4x^2 - 2$ then use the Rational Roots Theorem (aka the Rational Zeros Theorem) to list all possible rational zeros for $R(x)$.

$\dfrac{\text{factors of } -2}{\text{factor of } 3} : \dfrac{\pm 1}{1}, \dfrac{\pm 2}{1}, \dfrac{\pm 1}{3}, \dfrac{\pm 2}{3}$

$$\boxed{1, -1, 2, -2, 1/3, -1/3, 2/3, -2/3}$$

Problem 8: (2pts) It is known that $P(x) = x^3 - 4x^2 - 7x + 10$ has real zeros which are integers. Factor $P(x)$ completely. *Hint: use the Rational Roots Theorem*

$10 = 1 \cdot 10 = 2 \cdot 5 \cdot 1 \Rightarrow \pm 2, \pm 5, \pm 10$ possible zeros.

$P(1) = 1 - 4 - 7 + 10 = 0 \quad \therefore (x-1)$ factors $P(x)$

$P(-1) = -1 - 4 + 7 + 10 \neq 0$

$P(2) = 8 - 16 - 14 + 10 \neq 0$

$P(-2) = -8 - 16 + 14 + 10 = 0 \quad \therefore (x+2)$ factors $P(x)$

$P(5) = 125 - 4(25) - 35 + 10 = 0 \quad \therefore (x-5)$ factors $P(x)$

$$\therefore \boxed{P(x) = (x-1)(x+2)(x-5)}$$

Problem 9: (2pts) Factor the following polynomials completely over the complex numbers.

(a.) $x^4 - 4x^3 + 5x^2$

$$= x^2(x^2 - 4x + 5)$$
$$= x^2((x-2)^2 + 1)$$
$$= \boxed{x^2(x - 2 + i)(x - 2 - i)}$$

(b.) $x^4 - 5x^2 - 6$

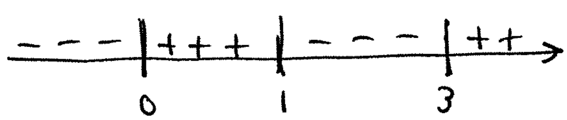
$$= (x^2 - 6)(x^2 + 1)$$
$$= \boxed{(x - \sqrt{6})(x + \sqrt{6})(x - i)(x + i)}$$

<u>Key Facts:</u>

$$(x - \alpha)^2 + \beta^2 = (x - \alpha + i\beta)(x - \alpha - i\beta)$$
$$(x - \alpha)^2 - \beta^2 = (x - \alpha + \beta)(x - \alpha - \beta)$$

Problem 10: (2pts) Solve $x^3 - 4x^2 + 3x \geq 0$. Write the answer in interval notation.

$$x(x^2 - 4x + 3) = x(x-1)(x-3) \geq 0$$

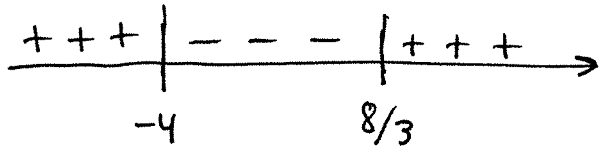


$$\Rightarrow \boxed{[0, 1] \cup [3, \infty)}$$

Problem 11: (2pts) Solve $\dfrac{x+4}{3x-8} \leq 0$. Write the answer in interval notation.

$x = -4$ makes numerator zero
$x = 8/3$ makes denominator zero

} places expression can change sign.



Note, cannot include $x = 8/3$ because $\div$ by zero,

$$\boxed{[-4, 8/3)}$$

Problem 12: (5pts) Consider the rational function $f(x) = \dfrac{2x^2 - 8x}{x^2 + 2x - 24}$. Find all vertical or horizontal asymptotes, as well as any holes in the graph. Graph the function carefully with each feature clearly labeled.

$\underline{y = 2}$ is the H.A. as $f(x) = \dfrac{2 - 8/x}{1 + 2/x - 24/x^2} \longrightarrow \dfrac{2-0}{1+0-0}$ as $x \to \pm\infty$.

Then consider,

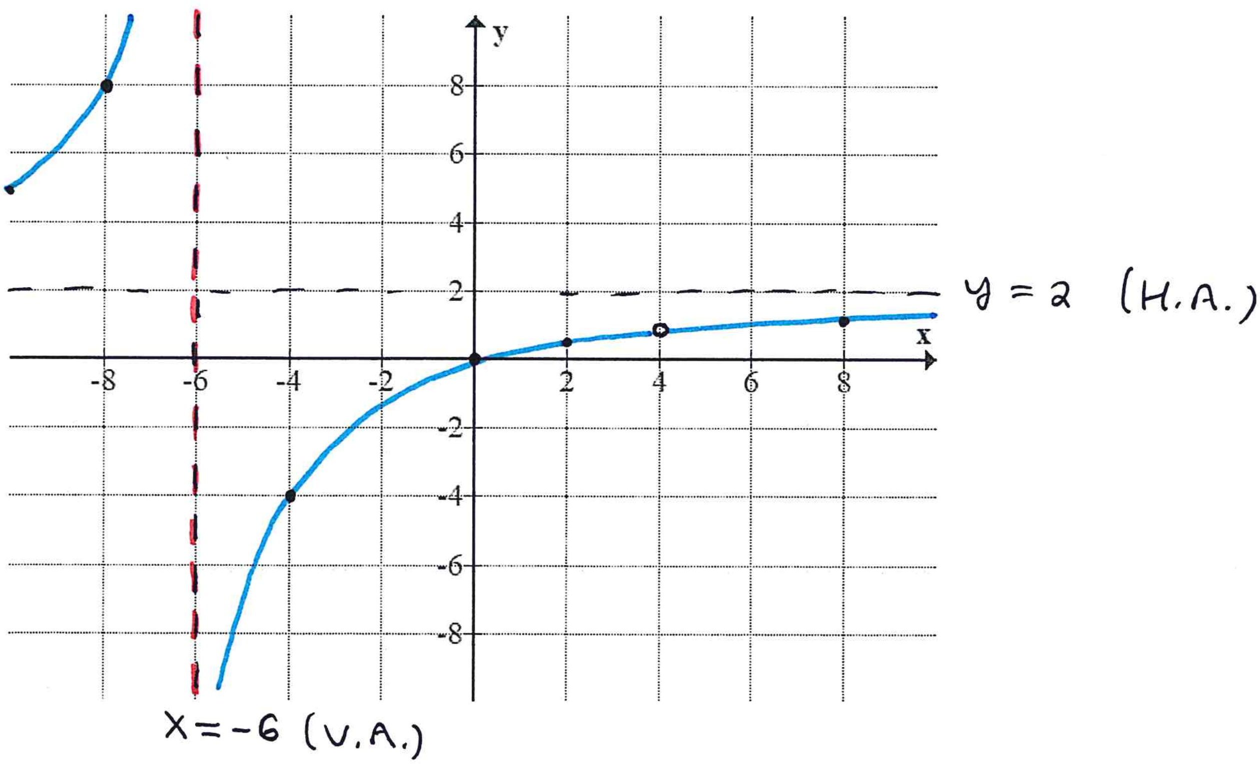
$$f(x) = \frac{2x(x-4)}{(x+6)(x-4)} = \underbrace{\frac{2x}{x+6}}_{f_{reduced}(x)} \quad \text{for } x \neq 4, -6.$$

Hole in graph at $(4, f_{red}(4)) = (4, \frac{2(4)}{4+6}) = (4, 0.8)$

V.A. at $x = -6$ and x-intercept at $x = 0$.

$f(-8) = \frac{-16}{-2} = 8$, $f(-4) = \frac{-8}{2} = -4$, $f(8) = \frac{16}{14} \cong 1.14$

$f(-10) = \frac{-20}{-4} = 5$ $\qquad f(1) = \frac{2}{7} \cong 0.29$



Problem 13: (1pts) Write the range of function in the previous problem in interval notation.

range $(f(x)) = \boxed{(-\infty, 0.8) \cup (0.8, 2) \cup (2, \infty)} = \{x \mid x \neq 0.8, 2\}$