

You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. For full credit please **BOX** your answers and show work. At least 150pts to earn here. Thanks!

Problem 1: (10pts) Find the equation of a line whose graph contains points $(2, -1)$ and $(0, 5)$.

Since $(0, 5)$ is on y -axis we find y -intercept of $b = 5$.

Then $y = mx + 5$ is eqⁿ of the line. Then $(2, -1)$ on line gives $-1 = m(2) + 5 \Rightarrow m = \frac{-6}{2} = -3$

$$\therefore \boxed{y = -3x + 5}$$

Problem 2: (10pt) Multiply the following expressions and collect like power terms to give your answer as a polynomial in standard form:

$$\begin{aligned} (x+4)^2(x^2-1) &= (x+4)(x+4)(x^2-1) \\ &= (x+4)[x^3+4x^2-x-4] \\ &= x[x^3+4x^2-x-4] + 4[x^3+4x^2-x-4] \\ &= x^4+4x^3-x^2-4x+4x^3+16x^2-4x-16 \\ &= \boxed{x^4+8x^3+15x^2-8x-16} \end{aligned}$$

Problem 3: (10pt) Assume $x, y > 0$ and use laws of algebra to determine A, B as indicated below:

$$\begin{aligned} x^A y^B &= \left(\frac{x^{-3} \sqrt{xy}}{(xy^3)^2} \right)^4 = \left(\frac{x^{-3} x^{1/2} y^{1/2}}{x^2 (y^3)^2} \right)^4 : (y^3)^2 = y^6, \frac{1}{y^6} = y^{-6} \\ &= \left(x^{-5} y^{-6} x^{1/2} y^{1/2} \right)^4 \\ &= (x^{-5})^4 (y^{-6})^4 x^{1/2} y^{1/2} \\ &= x^{-20} y^{-24} x^2 y^2 \\ &= x^{-18} y^{-22} \end{aligned}$$

$$\Rightarrow \boxed{A = -18} \quad \& \quad \boxed{B = -22}$$

Problem 4: (10pt) Solve $|2x + 3| + 2 = 13$.

$$|2x + 3| = 11$$

$$2x + 3 = \pm 11$$

$$2x = -3 \pm 11$$

$$x = \frac{-3 \pm 11}{2} = \frac{-3-11}{2} \text{ or } \frac{-3+11}{2} \Rightarrow \boxed{x = -7} \text{ or } \boxed{x = 4}$$

Problem 5: (20pt) Factor each $f(x)$ given below completely over \mathbb{R} :

(a.) $f(x) = x^3 - 9x^2 + 20x$

$$= x(x^2 - 9x + 20)$$

$$= \boxed{x(x-4)(x-5)}$$

(b.) $f(x) = x^4 - 13x^2 + 36$

$$= (x^2 - 4)(x^2 - 9)$$

$$= \boxed{(x+2)(x-2)(x+3)(x-3)}$$

Problem 6: (10pt) Solve $|4 - 3x| < 10$ and write your answer in interval notation.

$$-10 < 4 - 3x < 10$$

$$-14 < -3x < 6$$

$$\frac{-14}{-3} > x > \frac{6}{-3}$$

$$\frac{14}{3} > x > -2 \longrightarrow -2 < x < \frac{14}{3}$$

$$\therefore \boxed{(-2, 14/3)}$$

Problem 7: (10pts) Use completing the square and algebra as needed to place the circle equation below into standard form. Find the center and radius of the circle.

$$x^2 - 14x + y^2 + 25y = 1$$

$$(x - 7)^2 - 49 + \left(y + \frac{25}{2}\right)^2 - \left(\frac{25}{2}\right)^2 = 1$$

$$\underbrace{(x - 7)^2 + \left(y + \frac{25}{2}\right)^2}_{\text{Standard form.}} = 1 + 49 + \frac{(25)^2}{4} = \frac{825}{4}$$

$$\boxed{(7, -25/2) \text{ is center and } R = \sqrt{\frac{825}{4}} \cong 14.36}$$

Problem 8: (30pt) For each quadratic polynomial $f(x)$ given below, complete the square and find all real or complex solutions of $f(x) = 0$:

(a.) $f(x) = 3x^2 + 12x + 15$,

$$= 3(x^2 + 4x + 5)$$

$$= \boxed{3((x+2)^2 + 1)}$$

$$\text{Then } f(x) = 3((x+2)^2 + 1) = 0$$

$$\Rightarrow (x+2)^2 = -1$$

$$\Rightarrow x+2 = \pm \sqrt{-1} = \pm i$$

$$\therefore \boxed{x = -2 \pm i}$$

(b.) $f(x) = x^2 - 6x - 2$.

$$= (x-3)^2 - 9 - 2$$

$$= \boxed{(x-3)^2 - 11}$$

$$\text{Then } f(x) = (x-3)^2 - 11 = 0$$

$$\Rightarrow (x-3)^2 = 11$$

$$\Rightarrow x-3 = \pm \sqrt{11}$$

$$\therefore \boxed{x = 3 \pm \sqrt{11}}$$

Problem 9: (10pt) Find real numbers a, b for which $a + ib = \frac{26}{2 + 3i}$.

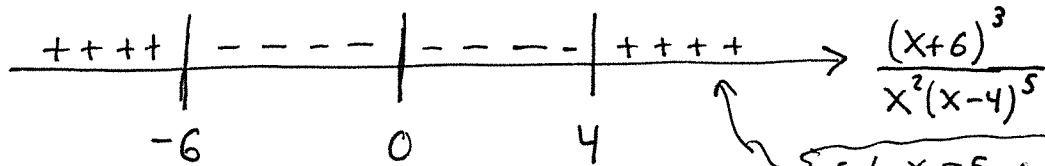
$$\begin{aligned}
 a + ib &= \frac{26(2 - 3i)}{(2 + 3i)(2 - 3i)} = \frac{26(2 - 3i)}{4 + 6i - 6i - 3i^2} \\
 &= \frac{26(2 - 3i)}{4 + 3^2} \\
 &= \frac{26(2 - 3i)}{13} = 2(2 - 3i) = 4 - 6i
 \end{aligned}$$

$a = 4$
 $b = -6$

Problem 10: (20pts) Solve the following inequality using an appropriate technique. Show your work and write the answer using interval notation (you might need to use \cup for union)

$$\frac{(x + 6)^3}{x^2(x - 4)^5} \geq 0$$

Algebraic critical #'s are $-6, 0, 4$. Consider then,



Need to exclude 4 because of division by zero.

$$(-\infty, -6] \cup (4, \infty)$$

set $x = 5$ get
 $\frac{11^3}{25(1)^5} > 0$
 so put (+++).

Problem 11: (10pts) Solve $\frac{1}{x+3} - \frac{2}{x-3} > 1$ and express your answer in interval notation using unions if appropriate.

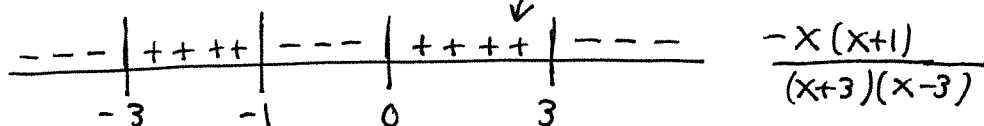
$$\frac{1}{x+3} - \frac{2}{x-3} - 1 > 0$$

$$\frac{x-3}{(x+3)(x-3)} - \frac{2(x+3)}{(x+3)(x-3)} - \frac{\overbrace{(x+3)(x-3)}^{x^2-9}}{(x+3)(x-3)} > 0$$

$$\frac{x-3 - 2(x+3) - (x^2-9)}{(x+3)(x-3)} > 0$$

$$\frac{-x^2 - x}{(x+3)(x-3)} = \frac{-x(x+1)}{(x+3)(x-3)} > 0$$

set $x = 1$ get
 $\frac{-1(2)}{4(-2)} > 0$



$$\therefore \boxed{(-3, -1) \cup (0, 3)}$$

Problem 12: (10pt Bonus) Let $P = (2, 0)$ and $Q = (8, 6)$ and $R = (3, 3)$ be vertices of a triangle. Find the area and perimeter of this triangle.

I'll take your solⁿ anytime before Test 2.
 Just bring me it sometime. Thanks!