You are allowed one page of notes and a calculator. No phones. More than 150pts to earn. Thanks!

Problem 1: (9pts) Suppose $f(2) = 20$ and $g(2) = 2$, and $g(20) = 11$. Calculate the following:

(a.) $(f+g)(2) =$ $f(2) + g(2) = 20 + 2 = \boxed{22}$

(b.) $(fg)(2) =$ $f(2)g(2) = 20 \cdot 2 = \boxed{40}$

(c.) $(g \circ f)(2) =$ $g(f(2)) = g(20) = \boxed{11}$

Problem 2: (15pts) Consider the graph $y = f(x)$ given below. Answer the following questions using interval notation where appropriate. Fill in the blanks:

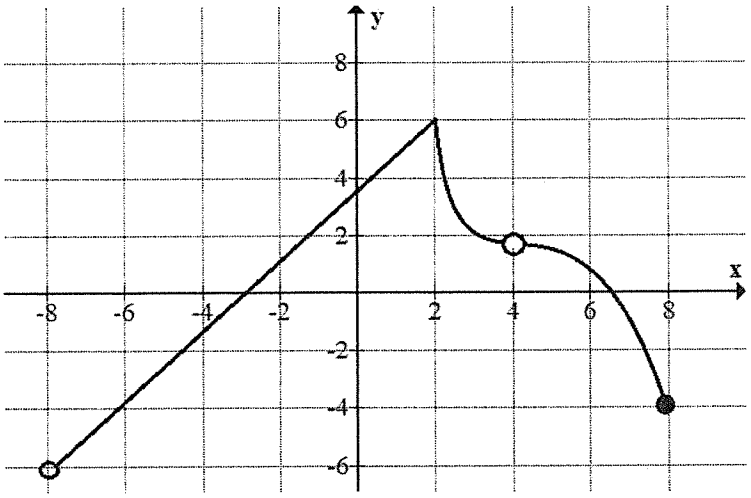
(a.) the domain of $f(x) =$ $(-8, 4) \cup (4, 8]$.

(b.) the range of $f(x) =$ $(-6, 6]$.

(c.) $f(4) =$ d.n.e.

(d.) $f(2) =$ 6

(e.) If $g(x) = x^2 + 4$ the calculate $(f \circ g)(2) =$ $f(g(2)) = f(2^2+4) = f(8) = \boxed{-4}$



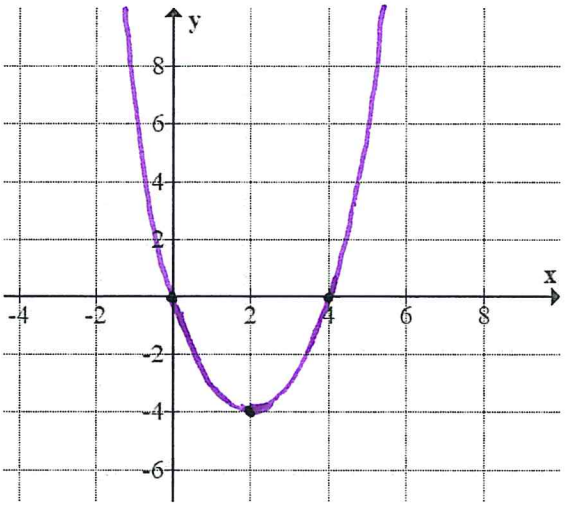
Problem 3: (6pts) Let $f(x) = \begin{cases} 2x^2 + 3 & : -2 < x < 1 \\ \sqrt{x+14} & : 1 \leq x \leq 3 \end{cases}$.

Given the function above, calculate:

(a.) $f(2) =$ $\sqrt{2+14} = \sqrt{16} = \boxed{4}$

(b.) $f(-1) =$ $2(-1)^2 + 3 = \boxed{5}$

Problem 4: Let $f(x) = x^2 - 4x$. Carefully graph $y = f(x)$ on the grid provided below (10pts). Also, find the range of the function and write it in interval notation (5pts).



$f(x) = x^2 - 4x = \underbrace{(x-2)^2 - 4}_{\text{vertex } (2,-4)} = y$

parabola opens up.

$f(4) = 0$
$f(0) = 0$ } $f(x) = x(x-4)$

$f(6) = 4^2 - 4 = 12$
$f(-2) = (-4)^2 - 4 = 12$ } out of picture.

$$\boxed{\text{range}(f) = [-4, \infty)}$$

Problem 5: (15pts) The difference quotient based at a for $f(x)$ is given by $\frac{f(a+h)-f(a)}{h}$ where $h \neq 0$. Calculate and simplify the difference quotient for $f(x) = \dfrac{1}{x+3}$.

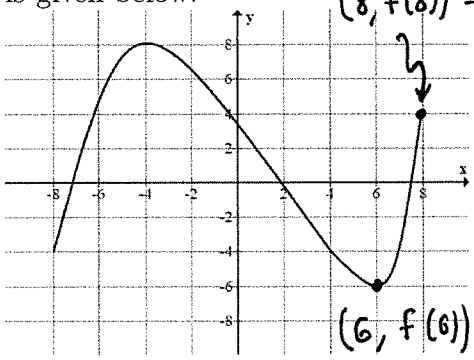
$$\frac{f(a+h)-f(a)}{h} = \frac{1}{h}\left[\frac{1}{a+h+3} - \frac{1}{a+3}\right]$$

$$= \frac{1}{h}\left[\frac{a+3-(a+h+3)}{(a+h+3)(a+3)}\right]$$

$$= \frac{1}{h}\left[\frac{-h}{(a+h+3)(a+3)}\right]$$

$$= \boxed{\frac{-1}{(a+h+3)(a+3)}}$$

Problem 6: (10pts) Find the average rate of change from $x = 6$ to $x = 8$ for the function whose graph is given below:



$(8, f(8)) = (8, 4)$

$(6, f(6)) = (6, -6)$

$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(6)}{8 - 6} = \frac{4 - (-6)}{2} = \frac{10}{2}$$

$$\boxed{\frac{\Delta y}{\Delta x} = 5}$$

Problem 7: (10pts) Given $f(x) = \sqrt{-x}$ and $g(x) = \sqrt{6 + 2x}$, calculate the formula for $(f + g)(x)$ and find the domain of $f + g$.

$f(x) = \sqrt{-x}$ needs $-x \geq 0$ $\Rightarrow$ $x \leq 0$ $\Rightarrow$ $\underline{\text{dom}(f) = (-\infty, 0]}$.

$g(x) = \sqrt{6 + 2x}$ need $6 + 2x \geq 0$

$2x \geq -6$

$x \geq -3$ $\Rightarrow$ $\underline{\text{dom}(g) = [-3, \infty)}$.

$\text{dom}(f + g) = \text{dom}(f) \cap \text{dom}(g) = \boxed{[-3, 0]}$.

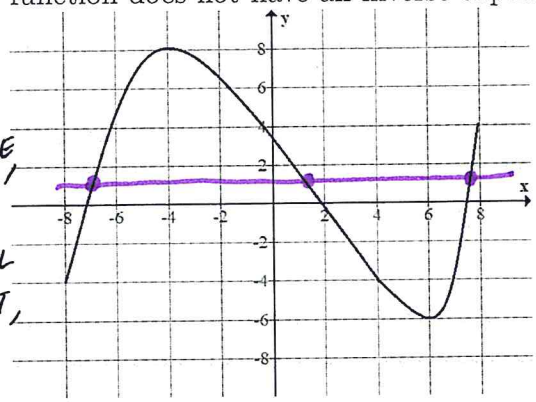
$(f + g)(x) = \boxed{\sqrt{-x} + \sqrt{6 + 2x}}$

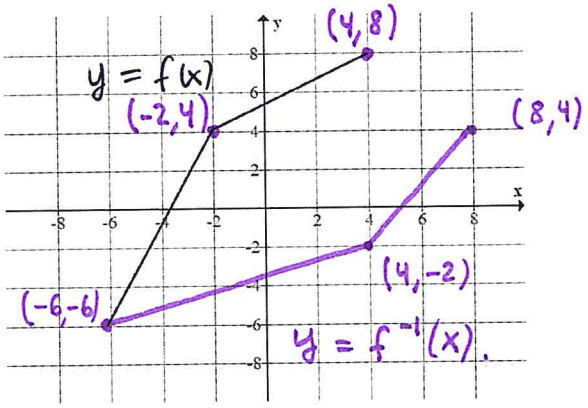
Problem 8: (10pts) For the functions given above, find the formula and domain for f/g.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{\sqrt{-x}}{\sqrt{6 + 2x}}} \quad \& \quad \boxed{\text{dom}\left(\frac{f}{g}\right) = (-3, 0]}$$

Problem 9: (12pts) If possible, graph the inverse function for each function graph below. If the function does not have an inverse explain why.

NOT INVERTIBLE, FAILS HORIZONTAL LINE TEST, NOT 1-1.





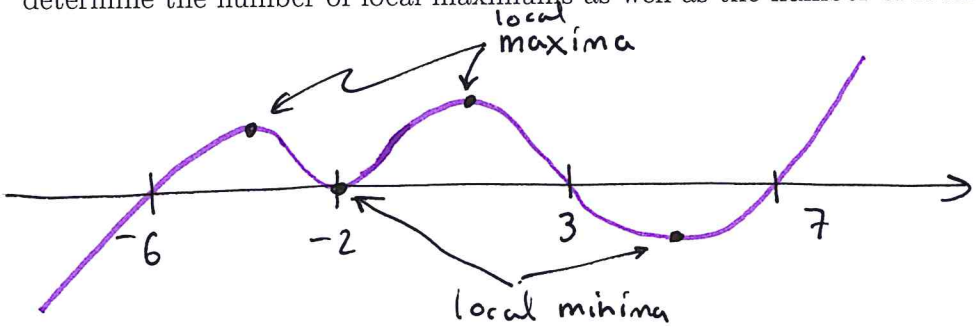
Problem 10: (18pts) Let $f(x) = (x+2)^3$ and $g(x) = \dfrac{1}{3-x}$. Find the formulas for:

(a.) $(f \circ g)(x) =$ $f(g(x)) = f\left(\frac{1}{3-x}\right) = \left[\frac{1}{3-x} + 2\right]^3$.

(b.) $(g \circ f)(x) =$ $g(f(x)) = g\left((x+2)^3\right) = \dfrac{1}{3-(x+2)^3}$.

(c.) $(f \circ f)(x) =$ $f(f(x)) = f\left((x+2)^3\right) = \left[(x+2)^3 + 2\right]^3$.

Problem 11: (10pts) Consider the graph $y = (x+6)(x+2)^2(x-3)(x-7)$. Sketch the graph and determine the number of local maximums as well as the number of local minimums.



2 local max's
2 local min's.

Problem 12: (10pts) Consider the graph of $yx - x^2 = yx^3$. Is this the graph of a function ?

$$yx - yx^3 = x^2$$
$$y(x - x^3) = x^2$$
$$y = \frac{x^2}{x - x^3}.$$

Yes, this is the graph of $f(x) = \frac{x^2}{x - x^3}$.

Problem 13: (12pts) Given the function $f(x) = 41 + \frac{x}{x-3}$ calculate the formula for $f^{-1}(y)$.

$$y = 41 + \frac{x}{x-3}$$
$$y - 41 = \frac{x}{x-3}$$
$$(x-3)(y-41) = x$$
$$xy - 41x - 3y + 123 = x$$
$$x(y - 41 - 1) = 3y - 123$$
$$x = \frac{3y - 123}{y - 42}$$
$$x = \boxed{f^{-1}(y) = \frac{3y - 123}{y - 42}}$$

Problem 14: (8pts) Find the domain and range $f(x)$ given in the previous problem.

$\text{dom}(f(x)) = \boxed{(-\infty, 3) \cup (3, \infty)}$ (must avoid $x = 3$)

$\text{range}(f(x)) = \text{dom}(f^{-1}(y)) = \boxed{(-\infty, 42) \cup (42, \infty)}$