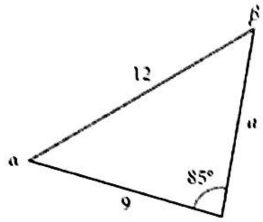
NAME ______________________________

MATH 114: FALL 2021 | **QUIZ 10 (25PTS)**

Show your work. Note, there is bonus possible if you get all correct.

Problem 1: (5pt) Find β in degrees for the following oblique triangle:



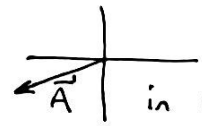
$$\frac{\sin\beta}{9} = \frac{\sin 85^\circ}{12} \quad (\text{Law of Sines})$$

$$\beta = \sin^{-1}\left(\frac{9\sin 85^\circ}{12}\right) = \boxed{48.34^\circ}$$

Problem 2: (5pt) Find the standard angle (in degrees) and magnitude of $\vec{A} = \langle -\sqrt{3}, -1 \rangle$

$$A = \sqrt{(-\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = \boxed{2}$$

$$\tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right) = 30^\circ \Rightarrow \theta = 180^\circ + 30^\circ$$



in Quad III.

$$\boxed{\theta = 210^\circ}$$

Problem 3: (5pt) Find the cartesian form of the complex number z with $|z| = 3$ and $\angle z = 240^o$

$$(240^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) = \frac{4\pi}{3} \text{ rad}$$

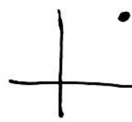
$$z = |z|e^{i\theta} = 3e^{\frac{4\pi i}{3}}$$

$$= 3\left(\cos\left(\tfrac{4\pi}{3}\right) + i\sin\left(\tfrac{4\pi}{3}\right)\right)$$

$$= 3\left(-\tfrac{1}{2} + i\left(-\tfrac{\sqrt{3}}{2}\right)\right) = \boxed{\frac{-3}{2} - \left(\frac{3\sqrt{3}}{2}\right)i}$$

Problem 4: (5pt) Find the polar form of the complex number $z = 2 + 2i\sqrt{3}$.

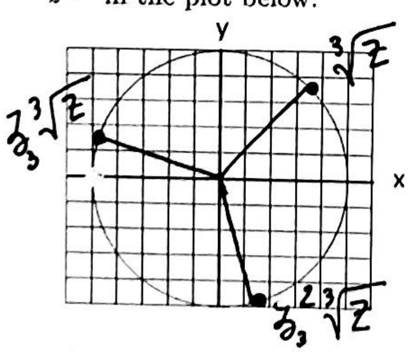
$$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4(1+3)} = \sqrt{16} = \boxed{4}$$



in Quad I

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = 60^\circ = \boxed{\frac{\pi}{3}\text{ rad.}} \rightarrow \boxed{z = 4e^{\pi i/3}}$$

Problem 5: (5pt) Let $z = 125e^{3\pi i/4}$. Find the Cartesian form of $\sqrt[3]{z}$ and plot all three elements of $z^{1/3}$ in the plot below:



$$\sqrt[3]{z} = \sqrt[3]{125}\, e^{\frac{3\pi i}{3\cdot 4}} = 5e^{\frac{\pi i}{4}}$$

$$\sqrt[3]{z} = 5\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right) = \frac{5}{\sqrt{2}}(1+i)$$

$$\boxed{\sqrt[3]{z} = \frac{5}{\sqrt{2}} + i\frac{5}{\sqrt{2}}}$$

$$z^{1/3} = \{\sqrt[3]{z},\ \underset{\text{rotate by }120^\circ}{\zeta_3\sqrt[3]{z}},\ \underset{\text{rotate by }240^\circ}{\zeta_3^2\sqrt[3]{z}}\} \quad // \quad \zeta_3 = e^{\frac{2\pi i}{3}} = \cos\left(\tfrac{2\pi}{3}\right) + i\sin\left(\tfrac{2\pi}{3}\right)$$

$$= \frac{-1}{2} + i\frac{\sqrt{3}}{2}$$