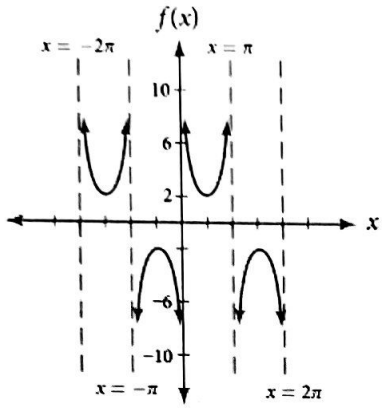
NAME _____

MATH 114: FALL 2021 — QUIZ 2 (10PTS)

You may use your homework solutions. I need to look at your class notes while you take this. You are allowed a 3x5 inch card of formulas. Thanks! 2pts per problem, hence 2pts bonus.

Problem 1: Find the formula for $f(x)$ given the graph below:
hint: the key is to understand where the vertical asymptotes come from and to note the graph does not hit y-values between $y = \pm 2$



$X = n\pi$ gives $\sin(n\pi) = 0$

$\Rightarrow$ this is $\csc(\theta)$ graph

$$\boxed{y = 2\csc(x)}$$

Problem 2: Use trigonometric identities to simplify the expression below:

$$\tan x \sin x + \sec x \cos^2 x = \left(\frac{\sin x}{\cos x}\right)\sin x + \frac{1}{\cos x}\cos^2 x$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \boxed{\sec(x)}$$

Problem 3: Use trigonometric identities to simplify the expression below:

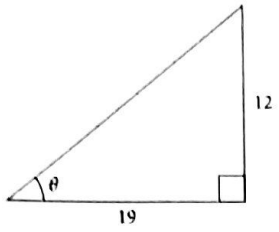
$$\frac{1-\cos^2 x}{\tan^2 x} + 2\sin^2 x = \frac{1-\cos^2 x}{\frac{\sin^2 x}{\cos^2 x}} + 2\sin^2 x \qquad \cos^2 x + \sin^2 x = 1$$

$$= \frac{(\cos^2 x)(\sin^2 x)}{\sin^2 x} + 2\sin^2 x$$

$$= \cos^2 x + \sin^2 x + \sin^2 x$$

$$= \boxed{1 + \sin^2 x}$$

Problem 4: Find the length of the hypotenuse and the angle θ in the triangle pictured below:

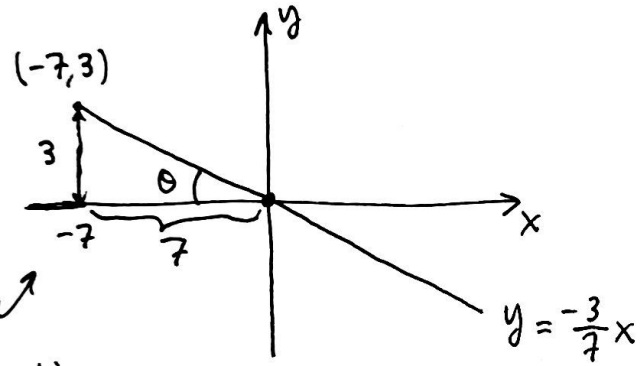


$c = \sqrt{12^2 + 19^2} = \boxed{\sqrt{505}}$ hypotenuse

$\tan\theta = \frac{12}{19} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{12}{19}\right) = 0.5633 \text{ rad.}$

$= 32.28°$

Problem 5: The line $y = -\frac{3}{7}x$ passes through the origin in the x, y-plane. What is the measure of the angle that the line makes with the negative x-axis ?



$\tan\theta = \frac{3}{7}$

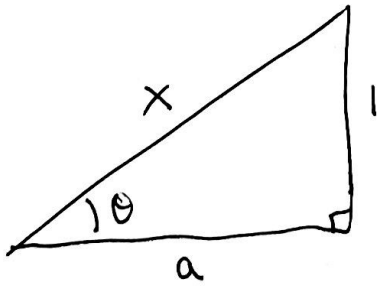
$\theta = \tan^{-1}\left(\frac{3}{7}\right)$

$\boxed{\theta = 0.405 \text{rad} = 23.2°}$

I plugged in $x = -7$ to make the triangle

Problem 6: Find the exact value of $\cos\underbrace{\left(\sin^{-1}\left(\frac{1}{x}\right)\right)}_{\theta}$ in terms of x with the help of a reference triangle.

$\sin^{-1}\left(\frac{1}{x}\right) = \theta \hookrightarrow \sin\theta = \frac{1}{x} = \frac{\text{opp}}{\text{hyp}}$



$a^2 + 1 = x^2$

$a^2 = x^2 - 1$

$a = \sqrt{x^2 - 1}$

(since $a > 0$)

Thus $\cos\theta = \frac{a}{x} = \boxed{\frac{\sqrt{x^2-1}}{x}}$