NAME ____

MATH 114: FALL 2021	TAKEHOME QUIZ 7 (15PTS)

Show your work. Note, there is bonus possible if you get all correct.

Problem 1: (4pts) Find the standard angle (in degrees) and magnitude of each of the following vectors:

(a.) $\vec{A} = \langle 2, 2 \rangle$
(b.) $\vec{B} = \langle -1, 3 \rangle$
(c.) $\vec{C} = \langle -2, -5 \rangle$
(d.) $\vec{D} = \langle 0, -6 \rangle$

Problem 2: (4pts) Find the vector $\vec{A}$ given the magnitude A and standard angle θ as follows:

(a.) $A = 2$, $\theta = 45^o$,
(b.) $A = 4$, $\theta = 120^o$,
(c.) $A = 10$, $\theta = 240^o$,
(d.) $A = 6$, $\theta = -30^o$,

Problem 3: (3pts) Find the angle between the vectors given below. Decide if the following vectors are parallel, perpendicular or neither.

(a.) $\vec{A} = \langle 1, 1, 1 \rangle$ and $\vec{B} = \langle 2, 0, -2 \rangle$,
(b.) $\vec{A} = \langle 1, 1 \rangle$ and $\vec{B} = \langle 3, 3 \rangle$
(c.) $\vec{A} = \langle 1, 3, 4 \rangle$ and $\vec{B} = \langle 5, 0, -3 \rangle$

Problem 4: (1pts) If $\vec{A}$ has magnitude 10 and $\theta = 30^o$ and $\vec{B} = \langle -3, 6 \rangle$ then find $\vec{A} + \vec{B}$.

Problem 5: (1pts) Suppose a vector $\vec{A}$ make angle $\alpha = 30^o$ with the positive x-axis and angle $\beta = 60^o$ with the positive y-axis and angle $\gamma = 45^o$ with the positive z-axis. Find the unit-vector in the direction of $\vec{A}$.

Problem 6: (1pts) Suppose $\vec{A}$ has length 6 and $\vec{A} \bullet \widehat{x} = 1$ and $\vec{A} \bullet \widehat{y} = -2$. Find all such $\vec{A}$.

Problem 7: (6pts) Calculate the cross-product and dot-product of the vectors given below:

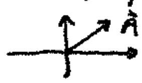
(a.) $\vec{A} = \langle 1, 3, 7 \rangle$ and $\vec{B} = \langle 2, 0, -2 \rangle$,
(b.) $\vec{A} = \langle 1, 0, 0 \rangle$ and $\vec{B} = \langle 3, 4, 5 \rangle$
(c.) $\vec{A} = \langle 1, 2, 2 \rangle$ and $\vec{B} = \langle 0, 1, -1 \rangle$

Problem 8: (2pts) Let $P = (1, 3, 0)$ and $Q = (5, 2, 1)$ and $R = (3, 4, 6)$. Find the interior angles and the area of the triangle PQR. *hint: use vectors*

Takehome Quiz 7 SOLUTION

P1 Find standard angle and magnitude,

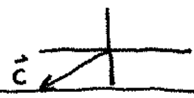
(a.) $\vec{A} = \langle 2, 2 \rangle$ has $A = \sqrt{2^2+2^2} = \boxed{\sqrt{8}}$ and $\Theta = \tan^{-1}\left(\frac{2}{2}\right) = \boxed{45^\circ}$



(b.) $\vec{B} = \langle -1, 3 \rangle$ has $B = \sqrt{1^2+3^2} = \boxed{\sqrt{10}}$

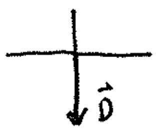
$\tan^{-1}\left(\frac{3}{-1}\right) = \tan^{-1}(-3) \cong -71.57^\circ \xrightarrow{\text{add } 180^\circ} \boxed{\Theta = 108.4^\circ}$

(c.) $\vec{C} = \langle -2, -5 \rangle$ has $C = \sqrt{4+25} = \boxed{\sqrt{29}}$



$\tan^{-1}\left(\frac{-5}{-2}\right) \cong 68.20^\circ \Rightarrow \Theta \cong 180^\circ + 68.2^\circ = \boxed{248.2^\circ}$

(d.) $\vec{D} = \langle 0, -6 \rangle$ has $D = \sqrt{0^2+36} = \boxed{6}$



$\tan^{-1}\left(\frac{-6}{0}\right)$ undefined! Note $\boxed{\Theta = 270^\circ}$

P2 Find $\vec{A}$ given magnitude A and standard angle Θ,
Notice $\vec{A} = \langle A\cos\Theta, A\sin\Theta \rangle$ guides our calculation,

(a.) $A = 2,\ \Theta = 45^\circ \Rightarrow \vec{A} = \langle 2\cos 45^\circ, 2\sin 45^\circ \rangle = \boxed{\langle \sqrt{2}, \sqrt{2} \rangle}$

(b.) $A = 4,\ \Theta = 120^\circ \Rightarrow \vec{A} = \langle 4\cos(120^\circ), 4\sin(120^\circ) \rangle = \boxed{\langle -2, 2\sqrt{3} \rangle}$

(c.) $A = 10,\ \Theta = 240^\circ \Rightarrow \vec{A} = \langle 10\cos(240^\circ), 10\sin(240^\circ) \rangle = \boxed{\langle -5, -5\sqrt{3} \rangle}$

(d.) $A = 6,\ \Theta = -30^\circ \Rightarrow \vec{A} = \langle 6\cos(-30^\circ), 6\sin(-30^\circ) \rangle = \boxed{\langle 3\sqrt{3}, -3 \rangle}$

$\boxed{P3}$ Find $\angle(\vec{A}, \vec{B})$ are $\vec{A}$ & $\vec{B}$ $\parallel$ or $\perp$ or neither?

(a.) $\vec{A} = \langle 1, 1, 1 \rangle$ and $\vec{B} = \langle 2, 0, -2 \rangle$

$\vec{A} \cdot \vec{B} = 1(2) + 1(0) + 1(-2) = 2 - 2 = 0 = AB\cos\theta \Rightarrow \boxed{\theta = 90^\circ}$

$\boxed{\vec{A} \perp \vec{B}}$

(b.) $\vec{A} = \langle 1, 1 \rangle$ and $\vec{B} = \langle 3, 3 \rangle$

$\vec{A} \cdot \vec{B} = 3 + 3 = 6$

$A = \sqrt{2}$ and $B = \sqrt{18}$

$\vec{A} \cdot \vec{B} = AB\cos\theta \Rightarrow 6 = \sqrt{2}\sqrt{18}\cos\theta$

$\Rightarrow 6 = \sqrt{36}\cos\theta = 6\cos\theta$

$\Rightarrow \cos\theta = 1 \quad \therefore \boxed{\theta = 0^\circ}, \boxed{\vec{A} \parallel \vec{B}}$

(c.) $\vec{A} = \langle 1, 3, 4 \rangle$ and $\vec{B} = \langle 5, 0, -3 \rangle$

$\vec{A} \cdot \vec{B} = 5 + 0 - 12 = -7 = AB\cos\theta = \sqrt{26}\sqrt{34}$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{26}\sqrt{34}}\right) \cong \boxed{103.62^\circ}$ neither $\parallel$ nor $\perp$ are the vectors $\vec{A}$ & $\vec{B}$.

$\boxed{P4}$ $\vec{A}$ has $A = 10$ and $\theta = 30^\circ$, $\vec{B} = \langle -3, 6 \rangle$

then $\vec{A} = \langle 10\cos 30^\circ, 10\sin 30^\circ \rangle = \langle 5\sqrt{3}, 5 \rangle$

$\vec{A} + \vec{B} = \langle 5\sqrt{3}, 5 \rangle + \langle -3, 6 \rangle$

$= \boxed{\langle 5\sqrt{3} - 3, 11 \rangle}$

$\boxed{P5}$ $\angle(\vec{A},\hat{x}) = 30°$, $\vec{A}\cdot\hat{x} = A\cos 30° = A_1 \Rightarrow \cos 30° = \frac{A_1}{A}$

$\angle(\vec{A},\hat{y}) = 60°$, $\vec{A}\cdot\hat{y} = A\cos 60° = A_2 \Rightarrow \cos 60° = \frac{A_2}{A}$

$\angle(\vec{A},\hat{z}) = 45°$, $\vec{A}\cdot\hat{z} = A\cos 45° = A_3 \Rightarrow \cos 45° = \frac{A_3}{A}$

where $\vec{A} = \langle A_1, A_2, A_3\rangle = A\hat{A}$

$$\hat{A} = \frac{1}{A}\vec{A} = \left\langle \frac{A_1}{A}, \frac{A_2}{A}, \frac{A_3}{A}\right\rangle = \langle \cos 30°, \cos 60°, \cos 45°\rangle$$

$$= \boxed{\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right\rangle}$$

Remark: $\|\hat{A}\| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}+\frac{1}{2}} = \sqrt{\frac{3}{2}} \neq 1$

OH NO !!! what does this mean?

It means my given angles are impossible. Sorry ☺

$\boxed{P6}$ $\vec{A}$ has $A = 6$, $\vec{A}\cdot\hat{x} = 1$ and $\vec{A}\cdot\hat{y} = -2$.

$\vec{A} = \langle A_1, A_2, A_3\rangle$ has $\vec{A}\cdot\hat{x} = A_1 = 1$ and $\vec{A}\cdot\hat{y} = A_2 = -2$

Hence, $\vec{A} = \langle 1, -2, A_3\rangle$

$$A = 6 = \sqrt{1^2 + (-2)^2 + (A_3)^2}$$

$$36 = 1 + 4 + A_3^2$$

$$A_3^2 = 31$$

$$A_3 = \pm\sqrt{31}$$

$$\therefore \boxed{\vec{A} = \langle 1, -2, \pm\sqrt{31}\rangle}$$

$\boxed{P7}$ Calculate $\vec{A}\times\vec{B}$ and $\vec{A}\cdot\vec{B}$

(a.) $\vec{A} = \langle 1, 3, 7\rangle$ and $\vec{B} = \langle 2, 0, -2\rangle$

$$\vec{A}\cdot\vec{B} = 1(2) + 3(0) + 7(-2) = \boxed{-12}$$

$$\vec{A}\times\vec{B} = \langle 3(-2) - 7(0), 7(2) + 1(2), 1(0) - 3\cdot 2\rangle$$

$$\vec{A}\times\vec{B} = \boxed{\langle -6, 16, -6\rangle}$$

(b.) $\vec{A} = \langle 1, 0, 0\rangle$ and $\vec{B} = \langle 3, 4, 5\rangle$

$$\vec{A}\cdot\vec{B} = 1(3) + 0(4) + 0(5) = \boxed{3}$$

$$\begin{aligned}\vec{A}\times\vec{B} &= \hat{x}\times(3\hat{x} + 14\hat{y} + 5\hat{z}) \\ &= 3\hat{x}\times\hat{x} + 4\hat{x}\times\hat{y} + 5\hat{x}\times\hat{z} \\ &= 4\hat{z} - 5\hat{y} \\ &= \boxed{\langle 0, -5, 4\rangle}\end{aligned}$$

(c.) $\vec{A} = \langle 1, 2, 2\rangle$ and $\vec{B} = \langle 0, 1, -1\rangle$

$$\vec{A}\cdot\vec{B} = 1(0) + 2(1) + 2(-1) = \boxed{0}$$

$$\vec{A}\times\vec{B} = \det\begin{bmatrix}\hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 2 \\ 0 & 1 & -1\end{bmatrix}$$

$$= \hat{x}(-2-2) - \hat{y}(-1-0) + \hat{z}(1-0)$$

$$= \boxed{\langle -4, 1, 1\rangle}$$

$\boxed{P8}$ was solved in Lecture on 11/17/21 in my Trigonometry Course.