NAME ______________________________

MATH 114: FALL 2021	TEST 2

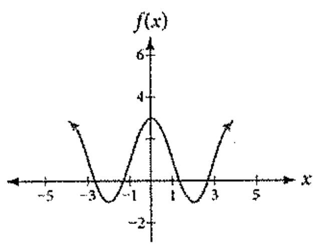
You may use the provided unit-circle and formula sheet. You are also allowed a 3x5 inch card of notes.

Problem 1: (12pts) Match each graph with the appropriate formula by filling in the blank below each graph with the letter corresponding the formula:

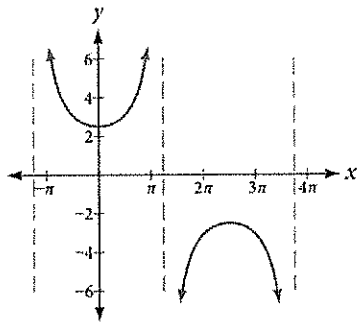
(A.) $y = 2\cos\left(\frac{\pi x}{2}\right) + 1$,

(B.) $y = 2.5\sec(0.4x)$, $= \dfrac{2.5}{\cos(0.4x)}$

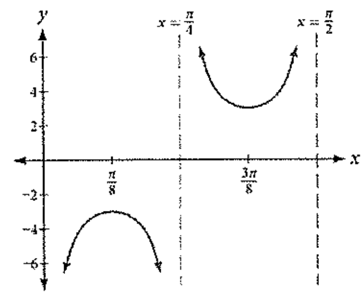
(C.) $y = -3\csc(4x) = \dfrac{-3}{\sin 4x}$



A.



B



C

Problem 2: (8pts) Given $\sin\theta = -1/2$ and $\cos\theta = \sqrt{3}/2$ find $\csc\theta$.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$$

Problem 3: (15pts) Use an appropriate identity to rewrite each of the following expressions:

(A.) $\cos 3x \,\cancel{\sin}^{\cos}\, 5x + \sin 3x \sin 5x =$ $\cancel{\sin(3x}$ $\cos(5x - 3x) = \boxed{\cos(2x)}$

(B.) $\sin 3x + \sin 7x =$ $2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right) = \boxed{2\sin(5x)\cos(2x)}$

(C.) $\cos 6x \cos 10x =$ $\frac{1}{2}\left(\cos(10x-6x) + \cos(10x+6x)\right) = \boxed{\frac{1}{2}\left(\cos 4x + \cos 16x\right)}$

Problem 4: (5pts) Suppose $\sin A = 0$ where $0 \le A \le \pi/2$ and $\cos B = 2/7$. Calculate $\cos(A+B)$.

If $\sin A = 0$ and $0 \le A \le \frac{\pi}{2}$ then $A = 0$ thus $\cos A = 1$

$\cos(A+B) = \cos A \cos B - \underbrace{\sin A}_{0} \sin B = 1 \cdot \cos B = \boxed{\frac{2}{7}}$

Problem 5: (6pts) Use trigonometric identities to rewrite the following expression in terms of $\cos\theta$:

$$1-\frac{\tan^2\theta}{\sec^2\theta} = 1 - \left[\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\right]\frac{1}{\left[\frac{1}{\cancel{\cos^2\theta}}\right]} = \underbrace{1-\sin^2\theta = \boxed{\cos^2\theta}}$$

$\sin^2\theta + \cos^2\theta = 1$

$1-\sin^2\theta = \cos^2\theta$

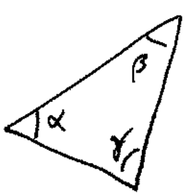
Problem 6: (6pts) Use trigonometric identities to simplify the following expression:

$$2\sin^2\theta + \cos(2\theta) = 2\left(\frac{1}{2}(1-\cos(2\theta))\right) + \cos(2\theta) \quad : \text{ by } 18$$

$$= 1 - \cos(2\theta) + \cos(2\theta)$$

$$= \boxed{1.}$$

Problem 7: (6pts) If α, β and γ are angles in the same triangle, then prove that $\cos(\alpha+\beta)+\cos\gamma = 0$.



$\alpha + \beta + \gamma = \pi \quad \therefore \alpha + \beta = \pi - \gamma$

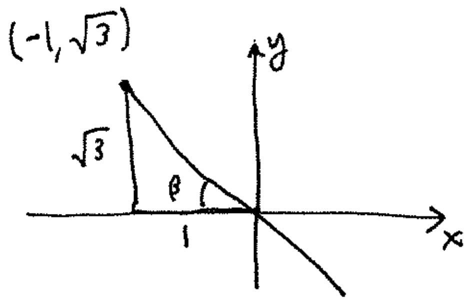
$$\cos(\alpha+\beta) + \cos\gamma = \cos(\pi - \gamma) + \cos\gamma \qquad \text{by } 11$$

$$= \cos\pi\cos(-\gamma) + \cancel{\sin\pi}\,\sin(-\gamma) + \cos\gamma$$

$$= -\cos\gamma + \cos\gamma \qquad : \cos(-\gamma) = \cos\gamma$$

$$= 0.$$

Problem 8: (6pts) The line $y = -x\sqrt{3}$ passes through the origin in the x,y-plane. What is the measure of the angle that the line makes with the negative x-axis ?



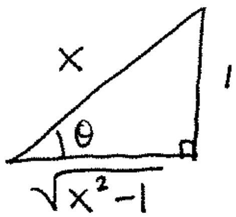
$$\tan\beta = \frac{\sqrt{3}}{1} = \frac{\sqrt{3}/2}{1/2} = \frac{\sin(\pi/3)}{\cos(\pi/3)}$$

$$\therefore \quad \beta = \tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$$

set $x = -1$
to obtain triangle
to measure β.

Problem 9: (9pts) Find the exact value of $\tan\left(\underbrace{\sin^{-1}\left(\frac{1}{x}\right)}_{\theta}\right)$ in terms of x with the help of a reference triangle.

$$\sin\theta = \frac{1}{x} = \frac{OPP}{HYP}$$



$$\Longrightarrow \quad \tan\theta = \boxed{\tan\left(\sin^{-1}\left(\frac{1}{x}\right)\right) = \frac{1}{\sqrt{x^2-1}}}$$

$$(adj)^2 + 1^2 = x^2$$

$$adj = \sqrt{x^2-1}$$

(choose + for length > 0)

Problem 10: (6pts) Write the range of each inverse function in interval notation on the blanks provided:

(A.) $range(\sin^{-1}) =$ $\underline{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

(B.) $range(\cos^{-1}) =$ $\underline{[0, \pi]}$

Problem 11: (15pts) Find a solution or state no solution exists.

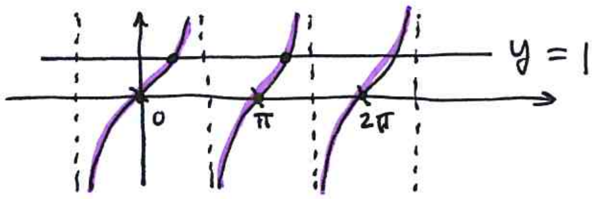
(A.) $\sin x = 0.3$ $\qquad x = \sin^{-1}(0.3) \cong \boxed{0.3047 \text{ rad} \cong 17.46^\circ}$

(B.) $\tan x = -1$ $\qquad x = \tan^{-1}(-1) = \boxed{-\frac{\pi}{4} \text{rad} = -45^\circ}$

(C.) $\cos x = 2$ $\qquad \boxed{\text{No sol}^{\underline{n}} \text{ exists}}$ $\quad -1 \le \cos x \le 2 \ \forall x \in \mathbb{R}$.

So $\cos x = 2$ is impossible $\forall x \in \mathbb{R}$.

Problem 12: (10pts) Find all solutions of $\tan x = 1$ for $0 \le x \le 2\pi$.

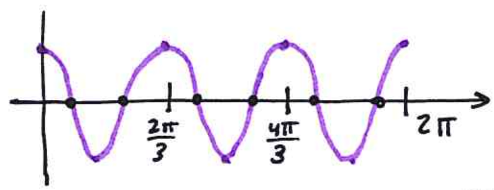


$\tan x = 1 \Rightarrow x = \frac{\pi}{4}$ (use $\tan^{-1}(1) = \frac{\pi}{4}$ if do not know already that $\tan\frac{\pi}{4} = 1$)

then $x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ also sol$^{\underline{n}}$ by π-periodicity of tangent

All other $x = n\pi + \frac{\pi}{4} \notin [0, 2\pi]$ for $n \neq 0, 1$ thus $\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

Problem 13: (10pts) Find all solutions of $\cos(3x) = 0$ for $0 \le x \le 2\pi$.



$3x = 2\pi$

$x = \frac{2\pi}{3}$: period of $\cos(3x)$

There are six solutions; $\underbrace{3x = \frac{\pi}{2}}_{x = \frac{\pi}{6}}$ and $\underbrace{3x = \frac{3\pi}{2}}_{x = \frac{\pi}{2}}$ are in $[0, 2\pi]$

then use $\frac{2\pi}{3}$-periodicity,

$x = \frac{\pi}{6}, \frac{\pi}{6} + \frac{2\pi}{3}, \frac{\pi}{6} + \frac{4\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

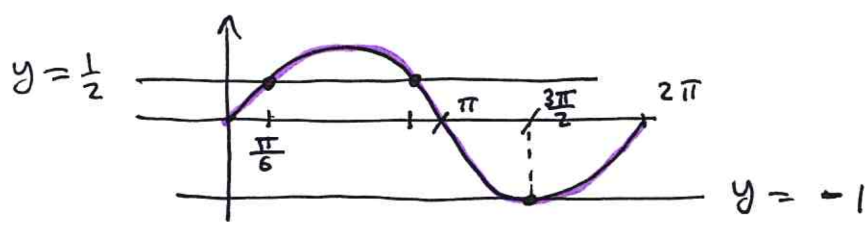
$x = \frac{\pi}{2}, \frac{\pi}{2} + \frac{2\pi}{3}, \frac{\pi}{2} + \frac{4\pi}{3} = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Problem 14: (10pts) Find all solutions of $2\sin^2 x + \sin x + 1 = 0$ for $x \in [0, 2\pi]$.

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\underbrace{\sin x = \frac{1}{2}}_{x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$ or $\underbrace{\sin x = -1}_{x = \frac{3\pi}{2}}$ (as explained)



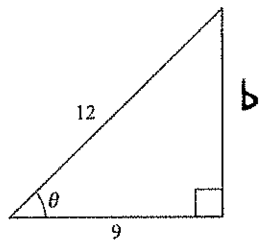
$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

By symmetry, $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ also gives $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

$\therefore \boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

Problem 15: (6pts) Find the **length** of the side opposite θ **and** find θ:



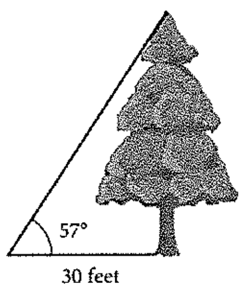
$$12^2 = 9^2 + b^2$$

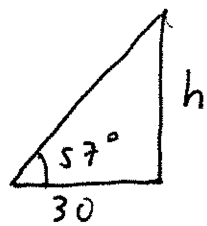
$$b = \sqrt{144 - 81} = \boxed{\sqrt{63}}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{63}}{9}\right) \cong \boxed{0.7227\text{rad} \cong 41.41^\circ}$$

$$\tan\theta = \frac{b}{9}$$

Problem 16: (5pts) Find the height of the tree.



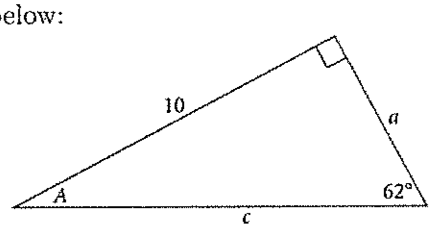


$$\tan 57^\circ = \frac{h}{30}$$

$$h = 30\tan 57^\circ$$

$$\boxed{h \cong 46.2\text{ ft}}$$

Problem 17: (10pts) Find the lengths a and c and find the measure of angle A of the triangle pictured below:



$$A + 90^\circ + 62^\circ = 180^\circ$$

$$\therefore \boxed{A = 28^\circ}$$

I used Law of Sines,

$$\frac{\sin(28^\circ)}{a} = \frac{\sin 90^\circ}{c} = \frac{\sin 62^\circ}{10}$$

$$a = 10\frac{\sin 28^\circ}{\sin 62^\circ} \cong \boxed{5.317}$$

$$c = 10\frac{\sin 90^\circ}{\sin 62^\circ} \cong \boxed{11.33}$$

(as a check, $a^2 + 10^2 \cong c^2$)

Alternate Path,

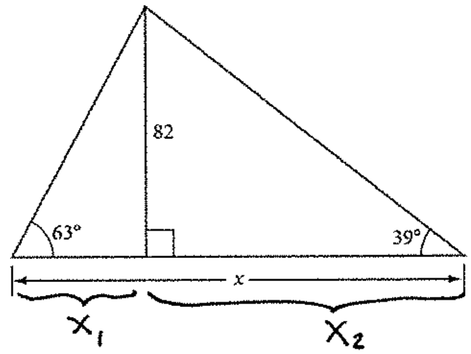
$$\tan 62^\circ = \frac{10}{a}$$

$$\cos 28^\circ = \frac{10}{c}$$

$$a = \frac{10}{\tan 62^\circ} \cong 5.317$$

$$c = \frac{10}{\cos 28^\circ} \cong 11.33$$

Problem 18: (10pts) Find x.

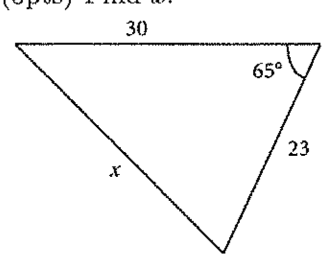


$$\tan 63° = \frac{82}{x_1} \quad \text{and} \quad \tan 39° = \frac{82}{x_2}$$

$$x_1 = \frac{82}{\tan 63°} \quad \text{and} \quad x_2 = \frac{82}{\tan 39°} \qquad (x_1 = 41.78 \ \& \ x_2 = 101.26)$$

$$\therefore \quad x = x_1 + x_2 = 82\left(\frac{1}{\tan 63°} + \frac{1}{\tan 39°}\right) = \boxed{143.04}$$

Problem 19: (5pts) Find x.



Law of Cosines,

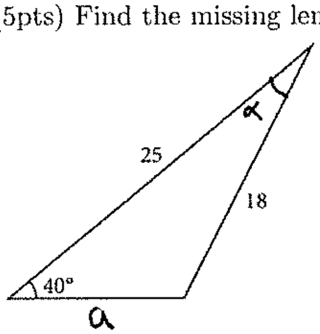
$$x^2 = 23^2 + 30^2 - 2(23)(30)\cos(65°)$$

$$x \cong \sqrt{1429 - 583.21}$$

$$x \cong \sqrt{845.79}$$

$$\boxed{x \cong 29.08}$$

Problem 20: (5pts) Find the missing length of the triangle below:



$$18^2 = a^2 + 25^2 - 2a(25)\cos(40)$$

$$a^2 - 38.30a + 301 = 0$$

$$a = \frac{38.3 \pm \sqrt{38.3^2 - 4(301)}}{2} \cong \boxed{27.26 \text{ or } 11.04}$$

from the picture,

$$\boxed{a \cong 11.04}$$

Identities Thus Far

(1) $\cos^2\theta + \sin^2\theta = 1$

(2) $1 + \tan^2\theta = \sec^2\theta$

(3) $\cot^2\theta + 1 = \csc^2\theta$

(4) $\cos(-\theta) = \cos\theta$

(5) $\sin(-\theta) = -\sin\theta$

(6) $\tan(-\theta) = -\tan\theta$

(7) $\sec(-\theta) = \sec\theta$

(8) $\csc(-\theta) = -\csc\theta$

(9) $\cot(-\theta) = -\cot\theta$

(10) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(11) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

(12) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(13) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(14) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

(15) $\sin(2\theta) = 2\sin\theta\cos\theta$

(16) $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

(17) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(18) $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta)) \xrightarrow[2\theta=\alpha]{\theta=\frac{\alpha}{2}} \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$ (21)

(19) $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta)) \xrightarrow[2\theta=\alpha]{\theta=\frac{\alpha}{2}} \sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$ (22)

(20) $\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)} \xrightarrow[2\theta=\alpha]{\theta=\frac{\alpha}{2}} \tan\left(\frac{\alpha}{2}\right) = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

(23)

(24) $\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$

(25) $\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$

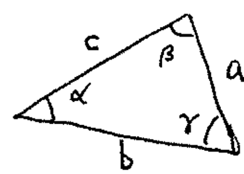
(26) $\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$

(27) $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

(28) $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

(29) $\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

(30) $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

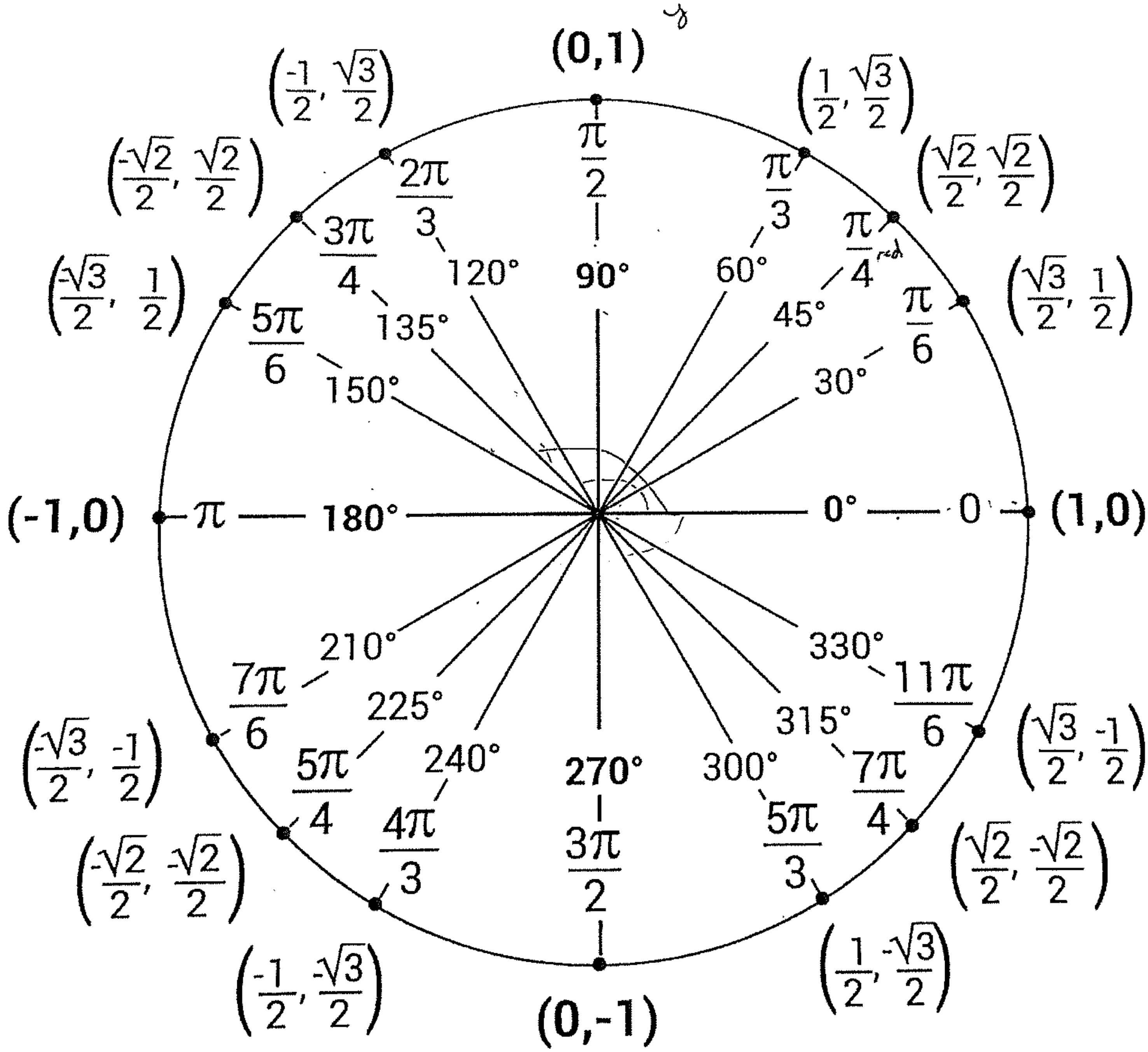


(31) $\boxed{\text{area } \Delta = \frac{1}{2}ab\sin\gamma}$

$a^2 + b^2 - 2ab\cos\gamma = c^2$
$a^2 + c^2 - 2ac\cos\beta = b^2$
$b^2 + c^2 - 2bc\cos\alpha = a^2$

Laws of Cosine (32)

$\underbrace{\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}}_{\text{Law of Sines}}$ (33)


(0,1)
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$ rad
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
120°
90°
60°
135°
45°
150°
30°
(-1,0)
π
180°
0°
0
(1,0)
210°
330°
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
225°
315°
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
240°
300°
270°
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
$\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
(0,-1)