Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

Problem 61 Your PRINTED NAME below indicates you have:

(a.) I have read Chapter 5 of Cook: ____________________.

Problem 62 Find the critical points of $f(x,y) = x^4 + y^4 - 16xy$

Problem 63 Consider $f(x,y) = \frac{2+x^2}{1+x^2+y^2}$. Use the geometric series to calculate the multivariate power series centered at $(0,0)$. Classify $(0,0)$ as a critical point or not, and, if it is a critical point determine if it is a min/max or saddle by examining the power series terms of second order. Also, graph the function and the second order multivariate Taylor polynomial to see how the power series locally approximates the function near the point of expansion (and to verify your conclusions from examining the coefficients).

Problem 64 Consider $f(x,y) = \sin(3\exp(-2x^2 - 2y^2))$. Find the multivariate Taylor series centered at $(0,0)$ up to second order. Explain why $(0,0)$ a critical point in view of the series you find. Also, classify the type of local extrema in view of the coefficients of the quadratic terms. Graph $z = f(x,y)$ to verify your claim.

Problem 65 Suppose $f(x,y,z) = 3 + 2(x-1) + 3y + 4z + (x-1)^2 + 3yz + 2(x-1)y + 4z^2 + \cdots$ is the multivariate power series of f centered at $(1,0,0)$ to second order. Calculate $\nabla f(1,0,0)$ and the values of the second derivatives $f_{xx}, f_{xy}, f_{xz}, f_{yy}, f_{yz}, f_{zz}$ at $(1,0,0)$.

Problem 66 Let $f(x,y) = (x-3)^2 + 2y^2 - 4(x-3) + 4y + 6$. Find the critical point of this function and use the second derivative test to determine if the point corresponds to a minimum, maximum or saddle point in the graph $z = f(x,y)$.

Problem 67 Let $f(x,y) = \tan^{-1}(xy)$. Find the critical point of this function and use the second derivative test to determine if the point corresponds to a minimum, maximum or saddle point in the graph $z = f(x,y)$.

Problem 68 Let $f(x,y) = 2xy\exp\left(\frac{-x^2-y^2}{2}\right)$. Find the critical points of this function and use the second derivative test to classify each points as a minimum, maximum or saddle point in the graph $z = f(x,y)$.

Problem 69 Find the extreme values of $f(x,y) = e^{xy}$ on the circle $x^2 + y^2 = 16$.

Problem 70 Find the extreme values of $f(x,y) = xy + x + y$ on the curve $x^2y^2 = 4$.

Problem 71 Find the minimum and maximum distances from the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ and the point (a,b,c). Hint: use the distance-squared function as the objective function and the ellipsoid as the constaint surface for a three-dimensional Lagrange multiplier problem.

Problem 72 Find the absolute extrema for $f(x, y) = x^2 + y^2 - 2y + 1$ on the partial disk D defined by $x^2 + y^2 \leq 4$ with $y \geq -1$.

Problem 73 Find the absolute extreme values of $f(x, y) = \sin(x^2/4 + y^2/2)$ on the triangle with vertices $(-1, 0), (1, 1)$ and $(1, -1)$.

Problem 74 We say $U \subseteq \mathbb{R}^n$ is path-connected iff any pair of points in U can be connected by a polygonal-path (this is a path made from stringing together finitely many line-segments one after the other).

(a) Show that if $\nabla f = 0$ on a path-connected set $U \subseteq \mathbb{R}^n$ then $f(\vec{x}) = c$ for each $\vec{x} \in U$

(b) Prove $\nabla g = \nabla h$ on a path-connected set $U \subseteq \mathbb{R}^n$ implies $g(\vec{x}) = h(\vec{x}) + c$ for all $\vec{x} \in U$ for some constant c.

For (a.) use the theorem from calculus I which states that if $f'(t) = 0$ for all t in a connected domain then $f = c$ on that domain. For (b.) simply consider $f = g - h$.

Problem 75 Prove the mean-value theorem for functions $\mathbb{R}^n \xrightarrow{f} \mathbb{R}$. In particular, show that if f is differentiable at each point of the line-segment connecting $\vec{P}$ and $\vec{Q}$ then there exists a point $\vec{C}$ on the line-segment $\overline{PQ}$ such that $\nabla f(\vec{C}) \bullet (\vec{Q} - \vec{P}) = f(\vec{Q}) - f(\vec{P})$.
Hint: parametrize the line-segment and construct a function on $\mathbb{R}$ to which you can apply the ordinary mean value theorem, use the multivariate chain-rule and win.

Bonus: An armored government agent decides to investigate a disproportionate use of electricity in a gated estate. Foolishly entering without a warrant he find himself at the mercy of Ron Swanson (at $(1, 0, 0)$), Dwight Schrute (at $(-1, 1, 0)$) and Kakashi (in a tree at $(1, 1, 3)$). Supposing Ron Swanson inflicts damage at a rate of 5 units inversely proportional from the square of his distance to the agent, and Dwight inflicts constant damage at a rate of 3 in a sphere of radius 2. If Kakashi inflicts a damage at a rate of 5 units directly proportional to the square of his distance from his location (because if you flee it only gets worse the further you run as he attacks you retreating) then where should you assume a defensive position as you call for back-up? What location minimizes your damage rate? Assume the ground is level and you have no jet-pack and/or antigravity devices.