

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

Problem 91 Your PRINTED NAME below indicates you have:

(a.) I have read §7.1 – 7.5 of Cook: _____.

Problem 92 Let f be a smooth function and \vec{G} a smooth vector field on some subset of \mathbb{R}^3 . Show that $\nabla \cdot (f\vec{G}) = \nabla f \cdot \vec{G} + f\nabla \cdot \vec{G}$ and $\nabla \times (f\vec{G}) = \nabla f \times \vec{G} + f\nabla \times \vec{G}$

Problem 93 Let $f(x, y, z) = x^2 + yz$ and $\vec{G}(x, y, z) = \langle x, z, z^2 \rangle$. Let $\vec{H} = f\vec{G}$ and calculate the curl and divergence of \vec{H} .

Problem 94 Let k be a constant and $\vec{F}(x, y, z) = \frac{k}{(x^2+y^2+z^2)^{3/2}} \langle x, y, z \rangle$. Calculate the curl and divergence of \vec{F} .

Problem 95 Let $\vec{B}(x, y, z) = \langle -y, x + z, 2 \rangle$. Find \vec{A} for which $\nabla \times \vec{A} = \vec{B}$. To my grader: sorry about this. There are infinitely many correct answers.

Problem 96 Let C be the curve with parametrization $x = e^t \cos(t)$, $y = e^t \sin(t)$ and $z = e^t$ for $0 \leq t \leq \frac{1}{2} \ln 3$. Assume the mass-density of C is uniform and find the center of mass of the wire as well as its arclength.

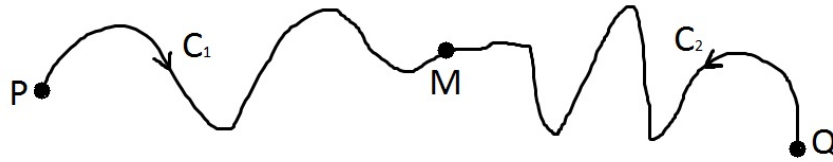
Problem 97 Let C be the curve with parametrization $x = 2 + 3t^2$ and $y = t^3 - 1$ for $0 \leq t \leq 1$. If $\vec{F}(x, y) = \langle 3x^2 + y, 3y^2 + x \rangle$ then calculate $\int_C \vec{F} \cdot d\vec{r}$.

Problem 98 Let $\vec{F}(x, y) = \langle x^3 + \sin(y), y^3 + \tan(x) \rangle$. Calculate the flux of \vec{F} through the unit-circle $x^2 + y^2 = 1$. I recommend you use the divergence form of Green's Theorem for this problem. Direct calculation of the flux seems less fun.

Problem 99 Let R be the rectangle with corners $(0, 0)$, $(1, 0)$, $(1, 3)$ and $(0, 3)$. Let ∂R be the CCW oriented boundary of R as the notation ∂R indicates. Calculate by Green's Theorem:

$$\oint_{\partial R} (\cosh(\sqrt{x} + 3) + y) dx + (3 + x^2) dy.$$

Problem 100 Suppose $\int_{C_1} \vec{F} \cdot d\vec{r} = 13$ and $\int_{C_2} \vec{F} \cdot d\vec{r} = -29$. Let C be the path from P to M to Q which follows the same set of points as C_1 and C_2 . Calculate $\int_C \vec{F} \cdot d\vec{r}$.



Problem 101 Let C be a path from $(0, 1)$ to $(2, 3)$. Calculate $\int_C (1 + 4x^3)dx + (2 - 2y^2)dy$.

Problem 102 Find the work done by the force $\vec{F}(x, y, z) = \langle y + x^2, x, z^3 \rangle$ on a mass as it moves from $(0, 0, 0)$ to $(1, 2, 3)$.

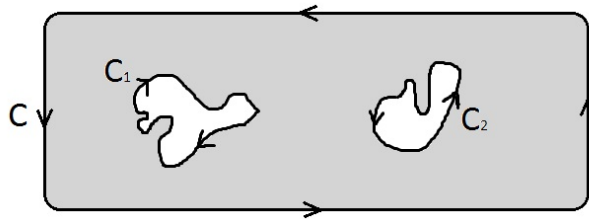
Problem 103 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\vec{F}(x, y) = \langle 2x - \sin(x + y^2), -2y \sin(x + y^2) \rangle$

(b.) $\vec{F}(x, y) = \langle -y, x \rangle$

(c.) $\vec{F}(y, z) = \langle z + y^2, y + z^3 \rangle$

Problem 104 Suppose we are given $\int_{C_1} \vec{F} \cdot d\vec{r} = -10$ and $\int_{C_2} \vec{F} \cdot d\vec{r} = 9$. Calculate $\int_C \vec{F} \cdot d\vec{r}$ given that \vec{F} is conservative (locally) on the domain between C and the curves C_1, C_2 . Is \vec{F} conservative on \mathbb{R}^2 ? Explain.



Problem 105 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants and suppose $\vec{F}_f = -b\vec{T}$ where v is your speed and b is a constant and \vec{T} is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by $\vec{F}_f + \vec{F}$ as you travel up the helix $\vec{r}(t) = \langle R \cos(t), R \sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.