P9 $f(x,y,z) = \dfrac{x\cos(y)}{1-z}$ find power series at $(0,0,0)$ to 2^{nd} order.

$$f(x,y,z) = x\left(1 - \tfrac{1}{2}y^2 + \cdots\right)\left(1 + z + z^2 + \cdots\right) = x\left(1 + z - \tfrac{1}{2}y^2 + z^2 + \cdots\right)$$
$$= \boxed{x + xz + \cdots}$$

P10 $f(x,y) = x^3 + y^3 - 3xy$

$$\nabla f = \langle 3x^2 - 3y,\ 3y^2 - 3x \rangle = \langle 0, 0 \rangle \quad \begin{cases} 3x^2 - 3y = 0 \\ 3y^2 - 3x = 0 \end{cases}$$

For critical points we need $x^2 = y$ and $y^2 = x \Rightarrow x^4 = x$

thus $x(x^3 - 1) = x(x-1)(x^2 + x + 1) = 0 \Rightarrow \underline{x = 0}$ or $\underline{x = 1}$

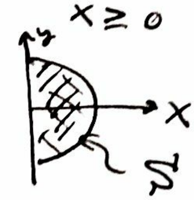
Hence obtain critical pts $\underline{(0,0) \text{ and } (1,1)}$.

Calculate $f_{xx} = 6x$, $f_{xy} = -3$, $f_{yy} = 6y$

thus $D = f_{xx}f_{yy} - f_{xy}^2 = 36xy - 9$

point	$f_{xx} = 6x$	$D = 36xy - 9$	$f(x,y)$	conclusion
$(0,0)$	0	$-9 < 0$	0	saddle
$(1,1)$	6	$27 > 0$	-1	minimum

P11 $f(x,y) = x^3 - 3x - y^2$ find extrema on $x^2 + y^2 \leq 9$, $x \geq 0$



① $\nabla f = \langle 3x^2 - 3,\ -2y \rangle = \langle 0, 0 \rangle$ for critical pts.

Need $y = 0$ and $3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

thus $(1,0)$ and $(-1,0)$ are potentially extremal

However, only $(1,0) \in S$. Calculate, $\underline{f(1,0) = 1 - 3 = -2}$.

② $\underbrace{g(t) = f(0,t)}_{-3 \leq t \leq 3 \text{ gives vertical line-segment}} = -t^2$ has $\dfrac{dg}{dt} = -2t = 0 \Rightarrow \underline{t = 0}$ critical

$$\left.\begin{aligned} f(0,3) &= -9 \\ f(0,-3) &= -9 \\ f(0,0) &= 0 \end{aligned}\right\} \text{ potential extremal values.}$$

③ $\underline{g(\theta) = f(3\cos\theta, 3\sin\theta)}$ has

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ parametrizes half-circle boundary

$$g = 27\cos^3\theta - 3\cos\theta - 9\sin^2\theta$$
$$\frac{dg}{d\theta} = -81\cos^2\theta\sin\theta + 3\sin\theta - 18\sin\theta\cos\theta$$
$$\frac{dg}{d\theta} = \sin\theta\left[-81\cos^2\theta + 3 - 18\cos\theta\right]$$

(3) Use $y = \pm\sqrt{9-x^2}$ to capture half-circle.

$$g_{\pm}(x) = f(x, \pm\sqrt{9-x^2}) = x^3 - 3x - (\pm\sqrt{9-x^2})^2$$

$$g_{\pm}(x) = x^3 - 3x - (9-x^2) = x^3 + x^2 - 3x - 9$$

$$\frac{dg_{\pm}}{dx} = 3x^2 + 2x - 3 = 0$$

$$x + \tfrac{2}{3}x - 1 = 0$$

$$\left(x + \tfrac{1}{3}\right)^2 - \tfrac{1}{9} - 1 = 0$$

$$\left(x + \tfrac{1}{3}\right)^2 = \frac{10}{9} \Rightarrow x = \frac{-1}{3} \pm \frac{\sqrt{10}}{3}$$

However, $0 \leq x \leq 3$ for the curve in question so choose (+)

$x = \dfrac{\sqrt{10}-1}{3}$ is potential critical point and

$y = \pm\sqrt{9 - \left(\frac{\sqrt{10}-1}{3}\right)^2}$ thus $\exists$ two critical pts. on half-circle to consider, they both give same value to f.

$$f\left(\frac{\sqrt{10}-1}{3}, \pm\sqrt{9-\left(\frac{\sqrt{10}-1}{3}\right)^2}\right) =$$

$$= g_{\pm}\Big(\underbrace{\tfrac{\sqrt{10}-1}{3}}_{\alpha \approx 0.7208}\Big) = \alpha^3 + \alpha^2 - 3\alpha - 9 \cong \boxed{-10.27}$$

minimum

Notice, $(3,0)$ is also a boundary pt. between $\pm\sqrt{\ }$ so we must consider it as well,

$$f(3,0) = 27 - 3(3) - 0^2 = \boxed{18}$$

maximum.

looking at the totallity of the analysis. (1), (2) and (3)