Show your work and box answers. Thanks and Enjoy! 415pts (and the 20pt bonus).

**Problem 1** (30pts) Let $\vec{F} = \langle x^2 + cos(yz), y^2, 3x\rangle$. Calculate $\nabla \bullet \vec{F}$ and $\nabla \times \vec{F}$.

**Problem 2** (30pts) Suppose the velocity of a ninja is given by $\vec{v} = \langle 2t, 3t^2, \sin(\pi t)\rangle$ at time $t$. If the position of the ninja is $(1, 0, 0)$ at time $t = 1$ then find the position and acceleration of the ninja as a function of time $t$.

**Problem 3** (30pts) Let $\vec{\gamma}(t) = \langle 3\cos t, 4t, 3\sin t\rangle$ for $t \geq 0$. Calculate the torsion of this curve.

**Problem 4** (30pts) Consider the ellipsoid $x^2 + y^2/4 + z^2/9 = 3$. Find the equation of the tangent plane to the ellipsoid at the point $(1, 2, 3)$

**Problem 5** (30pts) Let $f(x, y) = x^3 + 2xy$. Find the direction(s) in which $f$ changes at rate 0 at $(1, 1)$.

**Problem 6** (30pts) Calculate $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2}\, dy\, dx$

**Problem 7** (30pts) Let $C$ be parametrized by $x = t^2$ and $y = t + 3$ for $0 \leq t \leq 1$.
Calculate $\displaystyle\int_C y^2 dx - x dy$.

**Problem 8** (30pts) Let $C$ be the CCW oriented square with vertices $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$.
Calculate $\displaystyle\int_C (\tan^2(x) - 2y^2)\, dx + 43x\, dy$.

**Problem 9** (30pts) Let $S_R$ be the outward oriented sphere of radius $R$. Let $\vec{F} = \langle x^3, y^3, z^3\rangle$. Calculate the flux of the vector field through the sphere. That is, calculate: $\int_{S_R} \vec{F} \bullet d\vec{S}$.

**Problem 10** (45pts) Let $\vec{F}(x, y, z) = \dfrac{1}{(x^2 + y^2 + z^2)^{3/2}}\langle x, y, z\rangle$ for $(x, y, z) \neq (0, 0, 0)$.

**(a.)** work out $\nabla \bullet \vec{F}$ and simplify your result carefully,

**(b.)** Calculate $\iint_{S_R} \vec{F} \bullet d\vec{S}$ where $S_R = \{(x, y, z) \mid x^2 + y^2 + z^2 = R^2\}$ oriented outwards.

**(c.)** Explain why you could not use the Divergence Theorem in the previous part.

**Problem 11** (50pts) Suppose $\vec{F}(x, y, z) = \langle -2y, 2x, z^3\rangle$ and let $H$ be the open hemisphere $x^2 + y^2 + z^2 = 1$ for $z \geq 0$ oriented in the direction of increasing $\rho$. Verify Stokes' Theorem by calculating both of the following directly:

**(a.)** Calculate $\int_{\partial H} \vec{F} \bullet d\vec{r}$,

**(b.)** Calculate $\iint_H (\nabla \times \vec{F}) \bullet d\vec{S}$.

**Problem 12** (30pts) Let $\vec{F} = \langle 4x^3 + yz, xz, xy\rangle$. Let $C$ be any smooth curve from $(1, 1, 1)$ to $(2, 3, 0)$. Calculate $\int_C \vec{F} \bullet d\vec{r}$.

**Problem 13** (Bonus)(20pts) Consider $\alpha = (4x^3 + yz)\, dx + xz\, dy + xy\, dz$. Calculate $d\alpha$ and explain its significance to the previous problem.