MH 408: Calculus of Higher Dimension:	Test 2

Please box your answer. Show work for full credit.

(1.) (10pts) Let $f(x,y) = \sin(x^2 y)$ calculate f_{xx} and f_{yy} and f_{xy}.

(2.) (10pts) Calculate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ (if it doesn't exist then explain why)

(3.) (20pts) Let $f(x,y) = x^2 + 4xy + 6y^2$. For $(a.)$ and/or $(b.)$ leave answer (if any) as unit-vector with explicit decimal components to two decimal places.
(a.) in what direction(s) does f change at rate 9 at $(2,0)$?
(b.) in what direction(s) is f constant at $(2,0)$?

(4.) (10pts) Let $f(x,y,z) = z\sin(x^2+y^2)$.
Calculate the rate of change of f in the $\langle 2,1,2\rangle$-direction at $\left(\frac{\sqrt{\pi}}{2}, \frac{\sqrt{\pi}}{2}, 17\right)$.

(5.) (10pts) Suppose $w = xy + yz$ and $x + y^3 + z = 1$. Calculate $\left(\frac{\partial w}{\partial x}\right)_y$.

(6.) (10pts) Let $\rho = \sqrt{x^2+y^2+z^2}$. Show $\nabla\left(\frac{-1}{\rho}\right) = \frac{1}{\rho^3}\langle x,y,z\rangle$

(7.) (10pts) Consider the graph $z = 3 + x^2 + y^2$. Find the equation of the tangent plane at $(-1,2,8)$.

(8.) (15pts) Consider the surface M given as the solution set of $y^2 + z^2/4 = 1$. Find the following:

(a.) the equation of the tangent plane to M at $(7,0,2)$

(b.) a parametrization of the tangent plane to M at $(7,0,2)$

(9.) (5pts) Plot the level curve $F(x,y) = x^2/9 + y^2/4 = 1$ and sketch the gradient vector field ∇F along the curve. The curve plot should use a scale in which each box has length one. The vector field length for ∇F is unimportant, but, the direction is critical.

(10.) (12pts) Let $\vec{r}(s,t) = \langle s^2,\ t,\ st\rangle$ be the parametrization of a surface M. Calculate the normal vector field $\vec{N}(s,t) = \frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t}$ and find the equation of the tangent plane to M at $(4,-1,2)$.

(11.) (3pts) What is the Cartesian equation of M from the last problem?

(12.) (5pts) Suppose $h_x(1,2) = 10$ and $h_y(1,2) = 3$. In addition, $x(u,v) = e^{3u} + \sin(\pi v)$ and $y(u,v) = uv^2 + v$. Let $g(u,v) = h(x(u,v),\ y(u,v))$ and calculate $\frac{\partial g}{\partial u}$ for $u = 0$ and $v = 2$.

(13.) (10pts) A giant has an ice cream cone with a cap of ice cream given by $4x^2 + 5y^2 + 7z^2 = 16$ for $z \geq 0$ (in feet). Suppose a ninja dog named Earl runs over the ice cream and at the point $(1,1,1)$ we measure both the dx/dt and dy/dt to be $3ft/s$. What is the speed of Earl at $(1,1,1)$?